THE EFFECT OF MISMATCH ON THE COLLAPSE STRENGTH OF
MACHINED HEMISPHERICAL SHELLS

by

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July 1970

Report 3383
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ABSTRACT

Eight 4-in.-diameter, machined aluminum hemispheres with an axially symmetric mismatch machined in at the 45-deg meridional angle were hydrostatically tested to determine the effect of mismatch on the collapse strength of spherical shells. Results are plotted in terms of nondimensional parameters and are compared with previous test results from machined aluminum hemispheres without mismatch.

ADMINISTRATIVE INFORMATION

The work described in this report was funded under Naval Ship Systems Command Subproject, SF 35 422 305, Task 1960.

INTRODUCTION

A problem often encountered in the fabrication of spherical shells is the misalignment of adjoining segments. Very little information is available to aid the designer in predicting the effect of such misalignment or mismatch on the collapse strength of spherical shells subjected to external pressure. Tests of segmented HY-80 hemispherical shells indicate that the effect of mismatch on collapse strength is more pronounced on less stable shells and diminishes as the shells become more stable. 1 The contribution of mismatch to the premature collapse of fabricated shells has been partially obscured by other factors such as out of roundness. Thus eight models having different margins of stability and degrees of mismatch were machined to isolate the effect of mismatch. This report discusses the construction, testing, and results of these eight models.

DESCRIPTION OF MODELS

Eight 4-in.-diameter hemispherical models were machined from 5 1/2-in.-diameter 7075-T6 aluminum bilstock for which Young's modulus was taken as 10.8 x 10^6 psi and Poisson's ratio as 0.3. Figure 1 shows details and geometries of the models.

1References are listed on page 30.
Uniaxial compression tests performed on specimens from a piece of 5 1/2-in.-diameter 7075-T6 barstock indicated a considerable variation in strength with location and orientation. Specimens oriented axially at the center, tangentially at the center, axially at the edge, and tangentially at the edge had compressive yield strengths of 76,400, 79,700, 89,100, and 81,900 psi, respectively. The decrease in yield strength at the center of the barstock was attributed to the inability of the heat treatment to penetrate to the center of the relatively thick barstock.

Models were designed to cover the entire range of shell stability, with mismatches of 10 to 20 percent of the shell thickness. The margin of stability of a shell is defined by the ratio of the elastic buckling pressure \( P_E \) to the inelastic buckling pressure \( P_3 \) with:

\[
P_3 = 0.84 \frac{E}{R_0} \left( \frac{h}{R_0} \right)^2 \text{ for Poisson's ratio of 0.3}
\]

\[
P_E = 0.84 \sqrt{\frac{E_s}{E_t}} \left( \frac{h}{R_0} \right)^2 = \sqrt{\frac{E_s}{E_t}} P_3
\]

where

- \( E \) is Young's modulus,
- \( h \) is the nominal shell thickness,
- \( R_0 \) is the nominal outside radius,
- \( E_s \) is the secant modulus,
- \( E_t \) is the tangent modulus,

\[\sqrt{\frac{E_s}{E_t}} \] is the plasticity reduction factor, and
- \( e \) is the amount of mismatch.

Model designations reflect their \( P_3/P_E \) and \( e/h \) ratios. For example 30.2 indicates a \( P_3/P_E \) of 3.0 and an \( e/h \) of 0.2.

Machining of each model proceeded through four distinct operations. Following roughing out operations, the inside was machined between 0- and 45-deg vertical angle from a horizontal plane to a 2,000-in. radius. The center was then offset downward by an amount calculated to give the desired mismatch and the inside was machined between 45 and 90 deg vertical angle, a total on a 2,000-in. radius. Based on measured values of mismatch, a final
shell thickness was specified to give the desired e/h. The outside surface of the hemisphere was machined to yield the specified thickness using the same two centers for their respective vertical angular segments. An externally stiffened cylinder designed on the basis of measured hemispherical thickness to provide membrane boundary conditions was machined into the barstock at the base of the hemispherical portion.

TEST PROCEDURE

Each model was instrumented with 28 strain gages. These were placed at 22 and 67 deg and in the immediate vicinity of the mismatch along 1 vertical generator. Meridional check gages were placed near the mismatch along a generator 90 deg circumferentially away from the first generator. The diagram in Figure 2 shows a typical gage location.

Models were static tested in oil in either the NSRL 5- or 13-in. tank. The test setup is shown in Figure 3.

Each model was subjected to several pressure loadings. The maximum pressure applied on the first run was 70 percent of the estimated collapse strength of the model. Maximum pressure was successively increased on subsequent runs to collapse of the model. The final pressure increment prior to collapse was generally less than 3 percent of the collapse pressure.

TEST RESULTS

Experimental collapse pressures for each model are presented in Table 1. Experimental strain sensitivities are shown in Figure 4 where strain sensitivity is defined by the slope of the initial linear portion of the applied pressure versus measured strain curve and is given in micro-inches per inch per pounds per square inch. Pressure-strain plots in the area of the mismatch are given in Figure 5. Figure 6 shows the models after collapse. The greater destruction of Model 15.2 was due to the lack of internal venting.
**TABLE 1**

Comparison of Theoretical and Experimental Buckling Pressures

<table>
<thead>
<tr>
<th>No</th>
<th>$P_3$</th>
<th>$P_{el}$</th>
<th>$P_{exp}$</th>
<th>$\frac{\sigma_y}{P_E}$</th>
<th>$\frac{P_{exp}}{P_E}$</th>
<th>$k$</th>
<th>$P_{pred} - k P_E$</th>
<th>Reduction in Collapse Pressure Percentage</th>
<th>$e/h$ (Meas.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10.1</td>
<td>1764</td>
<td>1640</td>
<td>1480</td>
<td>1.09</td>
<td>0.902</td>
<td>0.94</td>
<td>1560</td>
<td>4</td>
<td>0.10</td>
</tr>
<tr>
<td>10.2</td>
<td>1710</td>
<td>935</td>
<td>935</td>
<td>0.90</td>
<td>0.767</td>
<td>0.90</td>
<td>820</td>
<td>7</td>
<td>0.10</td>
</tr>
<tr>
<td>15.1</td>
<td>946</td>
<td>2800</td>
<td>2700</td>
<td>1.71</td>
<td>0.934</td>
<td>1.00</td>
<td>2690</td>
<td>7</td>
<td>0.10</td>
</tr>
<tr>
<td>15.2</td>
<td>520</td>
<td>3090</td>
<td>2675</td>
<td>1.19</td>
<td>0.856</td>
<td>1.00</td>
<td>3090</td>
<td>13</td>
<td>0.10</td>
</tr>
<tr>
<td>20.1</td>
<td>422</td>
<td>4220</td>
<td>1900</td>
<td>2.26</td>
<td>0.924</td>
<td>1.01</td>
<td>4250</td>
<td>8</td>
<td>0.05</td>
</tr>
<tr>
<td>20.2</td>
<td>946</td>
<td>4160</td>
<td>345</td>
<td>2.23</td>
<td>0.852</td>
<td>1.01</td>
<td>4200</td>
<td>16</td>
<td>0.20</td>
</tr>
<tr>
<td>30.1</td>
<td>18,635</td>
<td>6670</td>
<td>5945</td>
<td>3.07</td>
<td>0.973</td>
<td>1.02</td>
<td>6190</td>
<td>4</td>
<td>0.10</td>
</tr>
<tr>
<td>30.2</td>
<td>71,020</td>
<td>6480</td>
<td>5900</td>
<td>1.24</td>
<td>0.910</td>
<td>1.02</td>
<td>6610</td>
<td>11</td>
<td>0.20</td>
</tr>
</tbody>
</table>

**Based on stress-strain curve shown in Figure 8.**

**Determined from curve for 7075-T6 aluminum hemispheres with ideal boundaries and no mismatch as found in Figure 7.**

**Reduction in collapse pressure = $\frac{P_{exp} - P_{pred}}{P_{exp}} \times 100$ percent.**
DISCUSSIONS

Nondimensionalized experimental buckling results for the eight models tested are tabulated in Table 1 and plotted in Figure 7. Variables are defined in the figure. A data point for a similar STS steel hemisphere with a 27-percent mismatch reported in Reference 1 is presented in Figure 7. For purposes of comparison, a curve representing the lower bound results of tests conducted by T.J. Kiernan of NSRDC on machined 7075-T6 aluminum hemispheres with ideal boundaries and no mismatch is included. This curve was used to determine predicted collapse pressures $P_{\text{pred}}$ for hemispheres with margins of stability the same as the mismatch models but without a mismatch. These values of $P_{\text{pred}}$ for perfect hemispheres are compared with experimental buckling pressures in Table 1 to determine the reduction in strength attributable to mismatch.

Present results as depicted in Figure 7 correlate well with the previously tested STS model. Note the general upward trend in the data with increasing margins of stability or $P_{\text{pred}}/P_F$.

The stress-strain curve for the barstock specimen with the lowest yield strength $\sigma_y$ was used to produce the plasticity reduction factor versus stress curve from which the plastic buckling pressures $P_F$ were calculated. This stress-strain curve is shown in Figure 8. It should be noted that if a stress-strain curve associated with one of the other specimens with a higher $\sigma_y$ had been used, the calculated values of $P_F$ would have been higher and this would have shifted the data points in Figure 7 downward and to the left.

A finite difference computer program to determine buckling of shells of revolution (BOSOR) was written by D. Bushnell of Lockheed Missiles and Space Company under contract to NSRDC. At present the analysis is confined to elastic shells. It is planned to eventually extend the analysis to the plastic range at which time the buckling of hemispheres with rotationally symmetric mismatches can be handled easily.

An elastic-plastic hybrid finite element stress analysis computer program is being developed at Brown University by H.D. Hulbert and P.V. Mareal under contract to NSRDC. This program uses a combination of shell
elements and triangular finite elements, and it is believed that it will be able to predict buckling of mismatched hemispheres. Although not a buckling analysis program as such, it is expected to detect the rapidly increasing displacements with steadily increasing applied pressure which characterizes the imminent buckling of certain imperfect structures.

STRAINS

Figure 9 compares experimental strain sensitivities with theoretical strain sensitivities calculated at 1 deg intervals along the shell using a finite difference computer program written by the author and based on the Penny elastic analysis of symmetric bending of thin shells of revolution. Boundary conditions were taken at 2 and 45 deg measured from the axis of revolution. Boundary conditions assumed were zero moment and shear at the 2 deg boundary and moment = $M_\alpha$, and radial shear = $Q_\alpha$ at the 45-deg boundary where from Reference 1:

\[
M = \frac{PR_0 e}{2},
\]

\[
M_\alpha = \frac{M}{2} = \frac{PR_0 e}{4},
\]

\[
H = \frac{M_\alpha}{R_0 \sin \alpha},
\]

\[
Q_\alpha = H \sin \alpha = \frac{N_\alpha}{R_0} = \frac{P}{4},
\]

\[
\lambda = \frac{4}{3(1-\nu^2)(k_0/h)} = 1.2854 \sqrt{\frac{k_0}{h}} \quad \text{for} \quad \nu = 0.3
\]

See Figure 10. Dimensions used were averages of measured dimensions as shown in Table 1. $P$ was taken as 0.0 psi. A unit external pressure was also applied along the shell surface.

The abscissa "position" of Figure 9 refers to angular orientation $\alpha$. Considering the severe strain gradient in the region of the mismatch, the agreement between the experimental points and the theoretical strain sensitivity curves was fairly good in most cases.
SUMMARY

Eight machined aluminum hemispheres with mismatches of up to 20 percent were hydrostatically tested to collapse. The reduction in strength due to mismatch ranged between 4 and 7 percent for models with a 10-percent mismatch and between 11 and 16 percent for models with a 20-percent mismatch. Measured elastic strain sensitivities near the mismatches agreed fairly well with those determined by thin shell theory.

ACKNOWLEDGMENTS

The major contributions of Mr. K. Nishida during the course of this project are gratefully acknowledged. The interest and suggestions of Messrs. M.A. Krunzke and T.J. Kiernan are appreciated. Thanks are also due Mr. J. Raines for his work in the conduct of model tests and data reduction.
Figure 3 - Test Setup
Figure 4 - Experimental Strain Sensitivities

(The strain sensitivity for each gage is given in parenthesis adjacent to the gage in $\text{um} / \text{in.} / \text{psi}$. The figure on the left is the sensitivity for the outside and the right for the inside. The minus sign indicates compressive strain. The asterisk indicates gages for which pressure strain plots are given on Figure 5.)

Figure 4a - Model 10.1

Figure 4b - Model 10.2
Figure 4c - Model 15.1

Figure 4d - Model 15.2
Figure 4g - Model 30.1

Figure 4h - Model 30.2
Figure 5 - Typical
(Positive strains indicated indicate compression)
Typical Pressure-Strain Plots

Positive strains indicate tension, negative strains indicate compression.

Figure 5a - Model 10.1
Fig. 5b Model 10.2

Strain in μin./in.
NOTE THAT CASES 105 & 106, 205 & 206 ARE Plotted IN REVERSE ORDER.
Figure Sf = Model 20.7
Figure 5h - Model 30.2
Figure 6 - Models After Collapse

Figure 6a - Model 10.1

Figure 6b - Model 10.2

Figure 6c - Model 15.1

Figure 6d - Model 15.2
Figure 7 - Nondimensional Plot of Test Results

\[ k = \frac{p_{\text{exp}}}{p_E} \]

\[ p_3 = 0.84 \frac{E}{R_0} (h/R_0)^2 \text{ for } \nu = 0.3 \]

\[ p_E = 0.84 \sqrt{E} \frac{E_t}{E} (h/R_0)^2 \]
Figure: Stress-Strain Curve for 5 1/2-Inch-Diameter 7075-T6 Aluminum Barstock at the Center and Normal to the Longitudinal Axis
Figure 9 - Experimental and Theoretical Strain Sensitivity Plots

Figure 9a - Model 10.1

Figure 9b - Model 10.2
Figure 9c - Model 20.1

Figure 9d - Model 20.2
Figure 9g - Model 30.1

Figure 9h - Model 30.2
Figure 10 - Sign Convention and Boundary Conditions for Finite Difference Stress Analysis
REFERENCES


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July 1970

PROJECT NO.

SH 35 422 305, Task 1960

IN TOTAL NO. OF PAGES

14

IN NO. OF SHEETS

7

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ABSTRACT

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