2010 IEEE International Conference on Robotics and Automation
Anchorage Convention District
May 3-8, 2010, Anchorage, Alaska, USA

Near-Optimal Autonomous Pursuit Evasion for Nonholonomic Wheeled Mobile Robot Subject to Wheel Slip

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***Abstract*—A new approach to autonomous pursuit evasion by a wheeled mobile robot in the presence of wheel slip is presented. Classical pursuit evasion problem, such as the Homicidal Chauffeur problem, considers the kinematic model of the pursuer and does not consider slip in its trajectory, and thus cannot predict a realistic pursuit evasion scenario. In this work we present a new dynamics-based approach to pursuit evasion problem in the presence of wheel slip. We first show how a feedback linearization controller can achieve capture with wheel slip. We then improve the capture time by designing a new extremum seeking controller that maximizes lateral traction force to effect a sharper but stable turn. The simulation results show the efficacy of our proposed control approach.**

I. Introduction

PURSUIT-evasion(P-E) problem has been studied since 1960s [1] as a differential game for two players. The problem is to find regions of initial conditions in game space that guarantee either capture (in capture region) or escape (in escape region) under optimal play, and determine corresponding optimal control strategies for both players subject to kinematic constraints. Especially when capture is guaranteed, the classical P-E problem gives an optimal solution for the pursuer to achieve capture with a given velocity and a curvature bound.

In more recent studies on P-E problems, various approaches have been developed to design controls for the pursuer and the evader. In [2], a randomized pursuer strategy is applied to locate an unpredictable evader and capture it in a visibility-based P-E problem. Dynamic programming is applied to find solution in a class of herding problem in [3], and in multi-player P-E problem in [4] where cumulant-based control is used. In [5] nonlinear model predictive controller is applied to an evasive UAV in an aerial P-E problem to help evasion. In [6] a graph theoretic approach is proposed to multi-player P-E problem. In [7] a time-optimal pursuit strategy was proposed in a P-E game and the pursuer takes worst analysis to capture the evader in a time-efficient and robust fashion even when the evader is intelligent.

Note that all these works assume kinematic constraints for the pursuer. For a wheeled mobile robot (WMR), this implies nonholonomic constraints that assume no-slipping condition, both longitudinally and laterally, at the wheels. However, for a realistic P-E problem with WMRs, wheel slip is inevitable. The classical game theoretic solution does not take wheel slip into account and thus cannot offer a solution that is valid in real-life WMR-based P-E problem where wheel slip is not negligible. So far there is no work, to our knowledge, that has analyzed the P-E problem subject to wheel slip, which is the focus of this paper.

In vehicle control, wheel slip determines the traction force upon which the maneuverability of a vehicle relies. Wheel slip is controlled to enhance the maneuverability of a vehicle, e.g., Antilock Braking System (ABS) control. The goal of ABS control is to maintain the longitudinal traction force at its maximum. Direct approaches drive the longitudinal traction force to its maximum using sliding mode-based extremum seeking control (ESC) [8][9][10] without knowing the optimal slip ratio or the analytic function of the longitudinal traction force, while indirect approaches [11][12][13][14] drive the wheel slip to its optimal value, derived from estimation or sensors, where the longitudinal traction force is maximal. *However, no work has been done in the literature to control the lateral traction force, which is in a non-actuated direction, to its maximum.*

In this paper, we explore new approaches to P-E problem subject to wheel slip. We focus on optimizing the pursuit process in a way that the capture time is minimized while the evader is under optimal evasion. Pursuit path can be split into curve and straight line segments. For constant speed pursuit, to minimize the time taken on the curve segment is to minimize the capture time. In this work, we exploit wheel slip to let the WMR take a sharp turn without being unstable such that the curve segment is minimized. In this method, we do not need to know the optimal slip angle, where the maximal lateral traction force occurs, or the analytic function of the lateral traction force. Instead we apply the ESC technique via sliding mode to maximize the lateral traction force of the pursuer in the curve segment such that the length of this segment is minimized. Sliding mode-based extremum seeking control was introduced by Korovin and Utkin [15][16], analyzed and applied by Ozguner and his coworkers on a variety of automotive problems, especially ABS design [8][9][10] and source tracing [17]. The model of a WMR on a slippery surface is an under-actuated system. The lateral direction is not actuated and the lateral traction force is indirectly controlled by the wheel torques. We apply ESC technique to control the traction force in the non-actuated direction to allow the WMR to take a sharp turn to optimize the capture of the evader.

This paper is organized as follows. In section II we briefly introduce the classical P-E problem where only kinematics of both players is considered. In section III, the full WMR model

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This work is partially supported by ARO grants DAAD19-02-1-0160 and ONR grand N00014-03-1-0052 and N00014-06-1-0146.

and the traction model between the wheel and the surface is introduced. As a first approach to solve the P-E problem with wheel slip, we present an input-output linearization-based feedback controller to compensate for the wheel slip to capture the evader in section IV. In section V, we present an improved controller that employs extremum seeking control technique to maximize the lateral traction force in the curve segment for the pursuer, in order to minimize the travel in this segment and thus minimize the capture time. In section VI we present the simulation results followed by a discussion and conclusion section (Section VII).

II. Description of P-E Problem

The P-E problem, which was initially considered as a differential game problem, is called the Homicidal Chauffeur problem in [1]. In this problem, the pursuer P moves at a fixed speed w_1, and its radius of curvature is bounded by a given quantity R. It steers by selecting the value of this curvature at each instant. The evader E moves with simple motion. Its speed w_2 ($w_2<w_1$) is fixed and it steers, at each instant, by choosing its direction of travel. Abrupt changes in this choice are allowed. Each player knows the other's kinematic states and relative location at each moment. Capture occurs when the distance PE $\leq l$, a given quantity. Herein there are two types of problems. The Game of Kind problem is to find precise conditions, values of R, l, w_1/w_2, which guarantee either capture or escape. The Game of Degree problem is to minimize the time to achieve capture. To take an example of this problem, we assume the evader is initially right behind the pursuer at a close distance, as located at e_0 and p_0 in Fig.1. The idea of pursuit is that, under optimal play, the pursuer first needs to go away from the evader, enlarge the distance in between until the pursuer reaches p_1 and the evader reaches e_1 simultaneously, then take a turn and go straight to the evader; while the evader at first follows the pursuer to e_1 and then escapes from it after the pursuer reaches p_1. More detail on deriving the optimal play strategies is omitted as it is elaborated in [1].

In this paper we will focus on the pursuit behavior for a WMR subject to wheel slip. One way to solve this problem is to include the dynamic model of the WMR with wheel slip and try to achieve a game theoretic solution for optimal play.

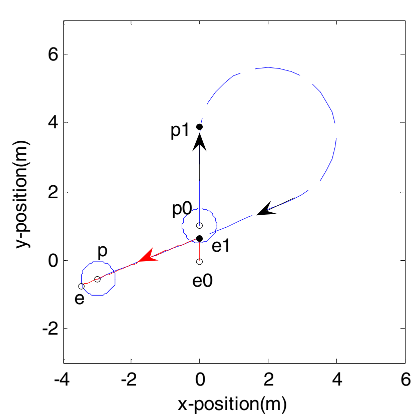

Fig. 1. Pursuit evasion paths: red line is evader's path; dashed blue curve is pursuer's path.

However, that will be extremely hard, if not impossible due to the additional complexities introduced by the dynamics, especially the variable slip of the WMR. Instead we assume the solution under the no-slip condition for optimal play as our starting point and design controllers that can negotiate slip: 1) following the nominal no-slip path (Section IV) and 2) by further reducing the radius of curvature by maximizing lateral traction force (Section V).

III. WMR Modeling with Wheel Slip

On slippery surfaces, the WMR is modeled as in Fig. 2, where P_c is the center of mass of the WMR, P_0 is the center of the wheel shaft, d is the distance from P_c to P_0, b is the distance from the center of each wheel to P_0. F_1 and F_2 are the longitudinal traction forces for $wheel_1$ and $wheel_2$, respectively. F_3 is the lateral traction force. To take into account the slip effect, dynamic model needs to be studied

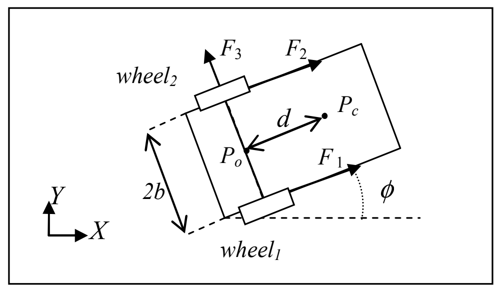

Fig. 2. WMR model on a slippery surface.

instead of kinematic model. The equations for the dynamic WMR model is derived from Newton's Law shown in Eq.(1).

$$\begin{cases} M\ddot{x}_c = (F_1+F_2)\cos\phi - F_3\sin\phi \\ M\ddot{y}_c = (F_1+F_2)\sin\phi + F_3\cos\phi \\ I\ddot{\phi} = (F_1-F_2)b - F_3 d \end{cases} \tag{1.1}$$

$$\begin{cases} I_w\ddot{\theta}_1 = \tau_1 - F_1 r \\ I_w\ddot{\theta}_2 = \tau_2 - F_2 r \end{cases} \tag{1.2}$$

where M is the robot mass, I is its moment of inertia, I_w is the moment of inertia of each wheel about the wheel axis, r is the wheel radius, ϕ is the orientation of the WMR, θ_i is the angular displacement of the i-th wheel, τ_i is the wheel torque applied to the i-th wheel. Equation (1.1) represents the entire WMR dynamics in the plane while (1.2) represents the spinning dynamics of the wheels.

The lateral and longitudinal traction forces are functions of slip angle (sa) and slip ratio (sr), respectively, and are modeled following the *Magic Formula* [18] in the literature. Slip angle and slip ratio are defined as

$$sr_i = \frac{r\dot{\theta}_i - v_i}{v_i}, \quad sa = \tan^{-1}\left(\frac{\dot{\eta}}{v}\right) \tag{2}$$

where v_i is the longitudinal speed of the center of the i-th wheel, $v = (v_1+v_2)/2$ is the forward velocity, $\dot{\eta}$ is the lateral speed of the center of each wheel. They satisfy the following nonholonomic constraints [19]

$$\dot{v}_1 = \dot{x}_c \cos\phi + \dot{y}_c \sin\phi + b\dot{\phi} \quad (3)$$

$$\dot{v}_2 = \dot{x}_c \cos\phi + \dot{y}_c \sin\phi - b\dot{\phi} \quad (4)$$

$$\dot{\eta} = \dot{y}_c \cos\phi - \dot{x}_c \sin\phi - d\dot{\phi} \quad (5)$$

Note that, unlike classical nonholonomic constraints of WMR, the above constraints allow both longitudinal and lateral slips.

The traction force between a wheel and a surface is modeled as

$$F = K_1 \sin\left(K_2 \tan^{-1}\left(SK_3 + K_4\left(\tan^{-1}(SK_3) - SK_3\right)\right)\right) + S_v \quad (6)$$

where S is a function of slip angle for lateral traction force and slip ratio for longitudinal traction force. All other variables K_i, i=1,…,4 and S_v are constants and determined from the curve fitting process of the empirical data. Fig. 3 shows an example of lateral traction forces with friction coefficient 0.7 and 0.3, respectively. The example of longitudinal traction force is omitted as its profile is similar to that of the lateral traction force.

IV. An Approach to P-E Problem Subject to Wheel Slip Using Input-Output Linearization

One approach to P-E problem on slippery surface is to develop an input-output linearization-based controller that takes into account the WMR model with slip in (1). With such a controller the pursuer is constrained on the nominal pursuit path indicated by the optimal play strategy in Section II, while the evader has the same kinematics and evasion strategy as before. For the straight line segments, we select the orientation ϕ and the forward velocity v of the pursuer as the outputs. For the curve segment, we select its angular velocity and forward velocity as the outputs, linearize the model, design a linear controller, and control the pursuer to track their desired values indicated by optimal play strategy updated at each moment. When the full model of the pursuer is introduced as in (1), the bound of the curvature, which is also the bound of the angular velocity of the wheels, is replaced by the bound of the wheel torque. We want to see how well the pursuer with wheel slip can follow the nominal pursuit path. With the control gains properly selected, simulation result is shown in Section VI. More details about input-output linearization technique and dynamic path following control applied on the WMR is omitted here as it can be found in [20][21][22].

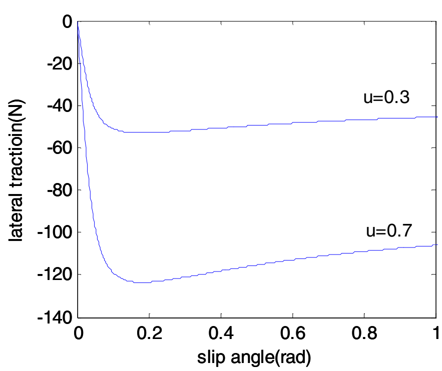

Fig. 3. Lateral traction for friction coefficients 0.7 and 0.3.

V. Near-Optimal P-E Problem Subject to Wheel Slip

In this paper, we define a near-optimal solution to the P-E problem in the sense that, the time it takes the pursuer to capture the evader at a constant speed is minimized. Since the pursuit path can be decomposed into straight line and curve segments, to minimize the curve segment is to minimize both the pursuit path and the capture time. In this section, we apply sliding mode-based ESC to maximize the lateral traction force of the pursuer when it is in the curve segment such that this segment is minimized. For straight line segments, we use the same input-output linearization technique to control the same outputs - orientation ϕ and forward velocity v of the pursuer - as in Section IV and do not discuss here. The evader has the same kinematics and evasion strategy as before.

A. Optimum Search Algorithm for Lateral Traction

Differentiating the lateral traction F_3 with respect to time along the trajectories of the system (1)-(5) we obtain

$$\frac{\mathrm{d}F_3}{\mathrm{d}t} = \frac{\partial F_3}{\partial (sa)} \frac{1}{v^2 + \dot{\eta}^2}[-\frac{dbv}{I}(F_1 - F_2) - \frac{\dot{\eta}}{M}(F_1 + F_2) + F_3(\frac{v}{M} + \frac{d^2 v}{I}) - \dot{\phi}(v^2 + \dot{\eta}^2 + d\dot{\phi}\dot{\eta})] + \frac{\partial F_3}{\partial t} \quad (7)$$

Define an error variable $e = F_3 - F_3^r$ where F_3^r is an upper bound of F_3. Then the dynamics for e is governed by

$$\frac{\mathrm{d}e}{\mathrm{d}t} = \frac{\partial F_3}{\partial (sa)} \frac{v}{v^2 + \dot{\eta}^2}[A(\dot{\eta}, \dot{\phi}, v, F_1, F_2) + Bu_1] + \frac{\partial F_3}{\partial t} \quad (8)$$

where

$$A(\dot{\eta}, \dot{\phi}, v, F_1, F_2) = -\frac{\dot{\eta}}{Mv}(F_1 + F_2) + F_3(\frac{1}{M} + \frac{d^2}{I}) - \dot{\phi}(v^2 + \dot{\eta}^2 + d\dot{\phi}\dot{\eta})/v, \quad (9)$$

$$B = -\frac{db}{I}, \quad (10)$$

and u_1 is the new control input defined as $u_1 = F_1 - F_2$.

We design the sliding surface as

$$s = e + \int_0^t \lambda e \mathrm{d}\tau, \quad (11)$$

where $\lambda > 0$. If s converges to a constant, the sliding motion satisfies

$$\frac{\mathrm{d}e}{\mathrm{d}t} + \lambda e \to 0, \quad (12)$$

and the lateral traction force can be made to its maximum with a proper selection of λ. To obtain the control law to let s converge to a constant, we rewrite (11) together with (8) as

$$\frac{\mathrm{d}s}{\mathrm{d}t} = \frac{\partial F_3}{\partial (sa)} \frac{v}{v^2 + \dot{\eta}^2}[A(\dot{\eta}, \dot{\phi}, v, F_1, F_2) + Bu_1] + \frac{\partial F_3}{\partial t} + \lambda e \cdot \quad (13)$$

Let $A = \overline{A} + \Delta A$ where $\overline{A}$ represent the nominal part of A whereas the unknown part ΔA is bounded by $|\Delta A| \le \delta A$.

Design the control law as

$$u_1 = -B^{-1}(\overline{A} + \gamma \Phi(s)), \quad (14)$$

where $\gamma = \delta A + N$ with N>0, and $\Phi(s) = \operatorname{sgn}\sin(2\pi s/\alpha)$, a periodic switching function [8] [9], which periodically search the traction force neighborhood to determine the control direction. This selection guarantees that s converges to $k\alpha$ for some integer k, which depends on the initial condition and F_3^r, if the following sliding mode existence condition is satisfied:

$$\left|\frac{\partial F_3}{\partial(sa)}\frac{v}{v^2+\dot{\eta}^2}(\delta A+N)\right| > \left|\frac{\partial F_3}{\partial(sa)}\frac{v}{v^2+\dot{\eta}^2}\Delta A+\frac{\partial F_3}{\partial t}+\lambda e\right|. \quad (15)$$

If it is assumed that the explicit dependence of traction on time is negligible, and keep in mind that $|\Delta A| \le \delta A$, the sliding mode existence condition turns into

$$\left|\frac{\partial F_3}{\partial(sa)}\right|\frac{v}{v^2+\dot{\eta}^2}N > \lambda|e|. \quad (16)$$

Thus in sliding mode, the lateral traction force will converge to F_3^r until it enters a region where the gradient is so small that the condition (16) cannot be satisfied. When (16) is not satisfied, the traction is close to its maximum and it will behaves arbitrarily. However, for a given F_3^r and λ, we can select a sufficiently large N such that this region around the maximum can be made arbitrarily small.

B. Forward Velocity Control

From (1)-(5) we obtain that the forward velocity is governed by

$$M\dot{v} = F_1 + F_2 + M\dot{\phi}(\dot{\eta}+d\dot{\phi}), \quad (17)$$

which we rewrite as

$$M\dot{v} = u_2 + M\dot{\phi}(\dot{\eta}+d\dot{\phi}), \quad (18)$$

where u_2 is the new control input defined as $u_2=F_1+F_2$.

We design sliding surface as

$$s = v - v_r, \quad (19)$$

where v_r is the desired speed. If s converges to zero, v will converge to v_r. The sliding surface is governed by

$$\dot{s} = \frac{u_2}{M} + \dot{\phi}(\dot{\eta}+d\dot{\phi}) = \frac{u_2}{M} + C(\dot{\phi},\dot{\eta}). \quad (20)$$

Let $C = \overline{C} + \Delta C$ where $\overline{C}$ represents the nominal part of C whereas the unknown part ΔC is bounded by $|\Delta C| \le \delta C$.

Design the control law as

$$u_2 = -M\overline{C} + Mk\operatorname{sgn}(s), \quad (21)$$

where $k = \delta C + \mu$ with $\mu > 0$, such that s converges to zero.

C. Lateral Traction Observer

As mentioned in subsection *A*, the realization of the ESC algorithm requires the knowledge of the lateral traction force. We assume this quantity cannot be measured directly, so we develop an observer which allows us to obtain lateral traction force using the measurements of WMR angular velocity $\dot{\phi}$ and wheel angular velocity $\dot{\theta}_i$. This observer is based on the equivalent control method, which has been used to develop observer for longitudinal traction force in ABS control in [8].

From (1) we obtain the dynamic equation

$$Ir\ddot{\phi} + I_w b(\ddot{\theta}_1 - \ddot{\theta}_2) = (\tau_1 - \tau_2)b - F_3 rd. \quad (22)$$

Now we define a new variable $\zeta = \dot{\phi} + \frac{I_w b}{Ir}(\dot{\theta}_1 - \dot{\theta}_2)$, which turns (24) into

$$Ir\dot{\zeta} = (\tau_1 - \tau_2)b - F_3 rd. \quad (23)$$

We define an estimate $\hat{\zeta}$ which satisfies

$$Ir\dot{\hat{\zeta}} = (\tau_1 - \tau_2)b - Vrd. \quad (24)$$

The function V is picked as

$$V = -N\operatorname{sgn}(\overline{\zeta}) \quad (25)$$

where $\overline{\zeta} = \zeta - \hat{\zeta}$ is a tracking error of ζ and N>0 is a sufficiently large constant.

Subtracting (24) from (23) we obtain

$$Ir\dot{\overline{\zeta}} = -rdN\operatorname{sgn}(\overline{\zeta}) - rdF_3. \quad (26)$$

If N is selected such that $N > \max\{|F_3|\}$, $\overline{\zeta}$ converges to the sliding surface $\overline{\zeta} = 0$. On sliding surface the equivalent value of variable $V = -N\operatorname{sgn}(\overline{\zeta})$ is equal to F_3

$$V_{eq} = F_3. \quad (27)$$

As shown in [8], the equivalent value of the high frequency switching signal can be obtained by applying a lowpass filter

$$H(s) = \frac{1}{T_f s + 1}, \quad (28)$$

where T_f is the constant which suppresses the high frequency signal. Since this chattering only occurs in the lateral traction force observer loop, it will not affect the entire system. The estimate of the lateral traction force out of the filter will be used in the ESC algorithm.

D. Longitudinal Traction Force Tracking

From previous sections *A* and *B*, we obtain desired F_1 and F_2 to control lateral traction force and forward velocity. Now, we design τ_i to enable F_i to track desired F_i using sliding mode control, which is omitted in this paper.

VI. Simulation Results for the P-E Problem

We present simulation results for the P-E problems studied in this paper. The simulation results for the three stages of the problem are shown below.

Stage 1: *Classical P-E Problem*

In this stage, both players have only kinematic constraints. As in Fig. 1 in section II, the paths of both players are shown in Cartesian space under given initial conditions. Given the starting positions of the pursuer and the evader as [0, 1] and [0, 0], respectively, it can be shown using game theoretic solution that when we select w_1=2m/s, w_2=0.5m/s, l=0.5m, R=2m, capture is guaranteed. The capture time is 8.7s.

Stage 2: *Feedback Control-based P-E Problem Subject to Wheel Slip*

This simulation is for the problem in the second part of section IV, where the pursuer has the full model as in (1) and is governed by an input-output linearization-based controller. M=17kg, I=0.6kg·m^2, I_w=0.0011kg·m^2, r=0.095m, b=0.24m, d=0.047m. The desired forward velocity for the pursuer is chosen to be 2m/s (same for all cases). Surface friction coefficient is 0.3 in this case. The P-E paths are illustrated in Fig. 4. We observe that the pursuer tries to follow the nominal pursuit path, while compensating the wheel slip, and eventually captures the evader. The capture time is 10.12s. Note that the shape of the capture path of the pursuer differs slightly from Fig. 1 due to wheel slip.

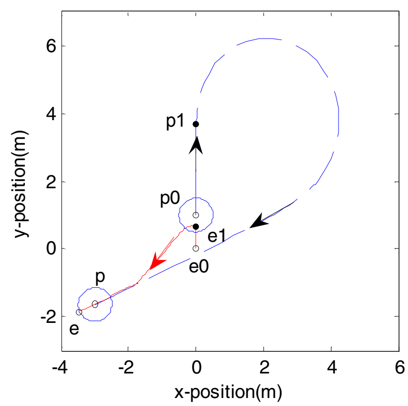

Fig. 4. Pursuit evasion paths where the pursuer is following the desired path via feedback control: red line is evader's path; dashed blue curve is pursuer's path.

Stage 3: *Near-Optimal P-E Problem Subject to Wheel Slip*

This simulation is for the problem in section V, where the pursuer has the full model as in (1) and is governed by an input-output linearization-based controller in straight line segments and by the optimal lateral traction force searching algorithm in curve segment. The pursuer has the same model parameters and desired velocity as in Stage 2. The friction coefficient is 0.3. The P-E paths are shown in Fig. 5. It is observed that the pursuer takes a sharp turn to capture the evader. The capture time is 6.8s.

Since the control technique we use in straight line segment is trivial, now we focus on the results in the curve segment, which corresponds to the time from 1.3s to 3.5s. When F_3^r, λ, and α are selected as -57N·m, 0.5, and 0.5, respectively, the actual lateral traction force moves to its maximum and stays in the small region around the maximum as shown in Fig. 6. The maximum of the lateral traction force can be

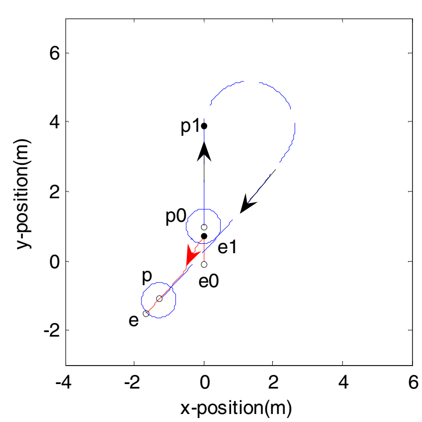

Fig. 5. Near-optimal pursuit evasion paths with pursuer on a slippery surface: red line is evader's path; dashed blue curve is pursuer's path.

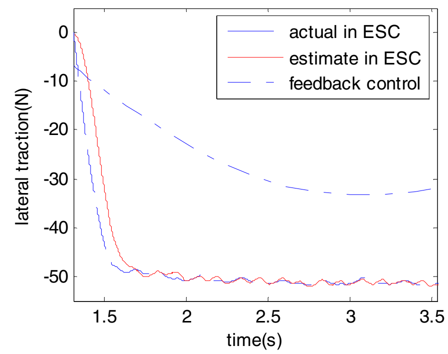

Fig. 6. The lateral traction force and its estimate from the observer for ESC in the curve segment, and the lateral traction force for output feedback control in the same time window.

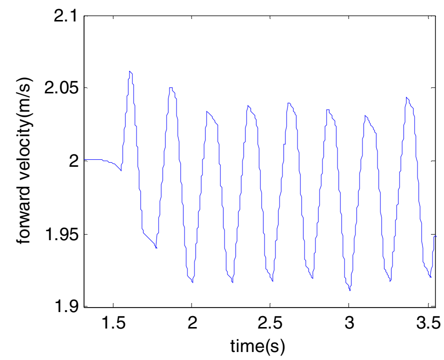

Fig. 7. WMR forward velocity in the curve section

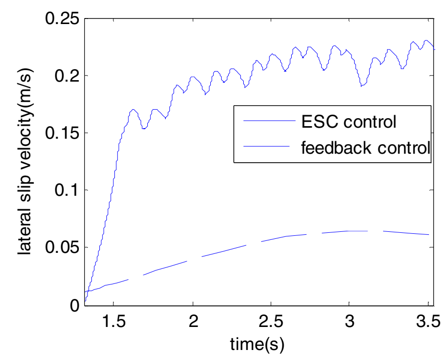

Fig. 8. Lateral slip velocity for ESC in the curve segment, and lateral slip velocity for output feedback control in the same time window.

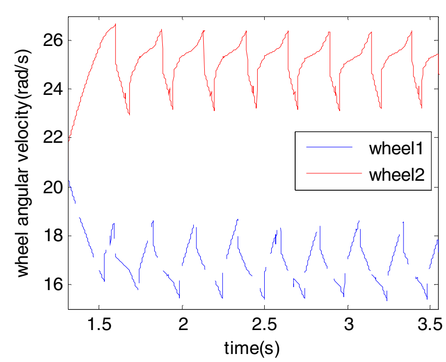

Fig. 9. Wheel angular velocity in the curve section.

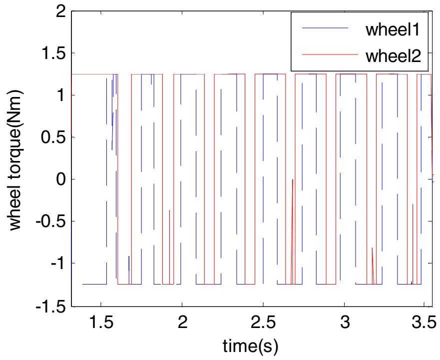

Fig. 10. Wheel torque in the curve section.

observed in Fig. 3. The output of the observer is a very good estimate of the force. However, the lateral traction force for output feedback control case in this duration is far from its maximum. Figure 7 is the forward velocity with chatter as it is controlled by sliding mode. In Fig. 8 the lateral slip velocity moves to its optimum and stays around it, which corresponds to optimal slip angle, while the lateral slip velocity for the output feedback control case is much less than the optimum. Fig. 9 is the angular velocity of each wheel and Fig. 10 is the applied torque for each wheel in which we set its bound at 1.24N·m.

Comparing the capture time for the four P-E problems in section II, IV, and V, respectively, in Table I, we see that, in the feedback control-based P-E, the capture time is longer than the classical P-E because the pursuer spends longer time in turning to deal with slip; while in the near-optimal P-E, the pursuer takes a turn as sharp as possible and captures the evader along the shortest path.

TABLE I
CAPTURE TIME FOR PURSUIT EVASION PROBLEMS

Problem type	Capture time(s)
Classical P-E with kinematics	8.7
Feedback control-based P-E	10.12
Near-optimal P-E	6.8

VII. Discussion and Conclusion

In this paper we approach the pursuit evasion problem considering both the dynamics of the WMR and the wheel slip as the WMR pursues the evader. We show that it is possible to exploit lateral slip to generate different curved paths during pursuit process and that may help the capture of the evader. We propose two controllers – one is a standard input-output linearization controller that works on a full dynamic model of the WMR with slip, and the other is a near-optimal extremum seeking controller that optimizes the lateral traction to minimize the radius of curvature – to address this problem. The simulation results show that the extremum seeking controller can give better performance to the pursuer.

Acknowledgment

The author gratefully acknowledges ARO grants DAAD 19-02-1-0160 and ONR grand N00014-03-1-0052 and N00014-06-1-0146 that partially supported this work.

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