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Analysis of order of redundancy relation for robust actuator fault detection

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ABSTRACT

This paper presents a new approach, called robust nonlinear analytic redundancy (RNLAR) technique to actuator fault detection for input-affine nonlinear multivariable dynamic systems. Robust fault detection is important because of the universal existence of model uncertainties and process disturbances in most systems. This paper characterizes the order of redundancy relation for nonlinear systems in terms of robustness. A theorem is proposed that state an increase in the order of redundancy relation increases the robustness in the sense of a performance index defined in this paper. Experimental results on a PUMA 560 robotic arm are presented to demonstrate the application of the theorem.

1. Introduction

During the last decade, as the applications of robots steadily expanded, there has been significant research activity in the area of robot reliability and fault tolerance (Visinsky, Cavalaro, & Walker, 1994). One way to address these needs is to design a fault tolerant control system (FTCS) for robotic systems. Generally, a FTCS consists of two major components: a fault detection and isolation (FDI) scheme, and a fault accommodation mechanism.

Considerable research effort has been invested in model-based fault detection methods since 1970s. Among them the parity relation-based schemes have been very successful. Some important survey papers in this area are given in Frank and Ding (1994, 1997), Gertler (1991), and Isermann (1984). The fundamental formulation of parity relation for linear systems is presented in Chow and Willsky (1984), which was based on analytic redundancy (AR) of the system. More detail is given in Gertler (1998). Robustness is an important aspect in the fault detection method. To address the robustness issue, Chow and Willsky (1984) have proposed an optimization method to select a parity vector from the parity space. This work was later extended by various researchers in Lou, Willsky, and Verghese (1986). Most recently in Han, Li, and Shah (2005) and Kwan and Xu (2004) the authors designed optimal primary residual that considered both the model-plant-mismatch (MPM) and process disturbances for linear systems.

The journey from linear AR residual generation methods to nonlinear analytic redundancy (NLAR) residual generation methods started with using linearized model of the nonlinear system to derive the AR residuals (Staroswiecki, Cassar, & Comtet-Varga, 1997; Zhirabok & Prebragenskaya, 1993). The AR concept was later extended to nonlinear systems without linearization. In Leuschen, Walker, and Cavallaro (2005) the authors proposed a NLAR scheme based on parity relation method.

It was pointed out in Lou et al. (1986) that the selection of the order of redundancy relation has an influence on the optimization performance. In fact, it is proved in Ding, Guo, and Jeinsch (1999) that increasing the order of redundancy relation leads to an increase in the dimension of the parity space, which in turn provides greater flexibility in residual generation as well as improves robustness. Note that the above-discussed conclusions regarding the increase in order of redundancy relation have been proven for linear systems. There are no equivalent results available in the literature for nonlinear systems. The objective of this paper is to extend the above results for nonlinear systems.

Recently, a new robust nonlinear analytic redundancy (RNLAR) technique for fault detection was developed (Halder & Sarkar, 2005, 2007) which accommodates both the MPM and process disturbances for nonlinear multivariable dynamic systems. In Halder and Sarkar (2005) it is pointed out that an increase in the order of redundancy relation increases the robustness of the nonlinear residuals. This result is compatible with its linear counter part as given in Lou et al. (1986). Experimental results using a PUMA 560 robotic arm are presented to demonstrate the usefulness of the theorem.

The paper is organized as follows. Section 2 presents the RNLAR residual generation. Section 3 gives the main theorem of this paper and provides the proof. The experimental results on

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PUMA 560 robotic arm to show the effectiveness of the increase order of redundancy are presented in Section 4. Summarization of the contributions is given in Section 5.

2. RNLAR residual formulation

Consider a multivariable input-affine nonlinear dynamic system of the form:

$$\dot{x} = f(x) + \sum_{i=1}^{q} g_i(x)u_i + d(x,u), \quad y = Cx + o \tag{1}$$

where the state x is defined on an open subset U of $\Re^n$; $u = [u_1 \ u_2 \ \ldots \ u_q]^T \in \Re^q$ is the input; $y \in \Re^m$ is the process output; C is the $m \times n$ output matrix; $d(x,u)$ represents an unmeasured deterministic process disturbance vector; o represents a Gaussian-distributed white noise vector. The functions $f, g_1, \ldots, g_q$ are $\Re^n$ valued smooth mappings defined on the open set U, and define $g = [g_1 \ g_2 \ \ldots \ g_q] \in \Re^{n \times q}$.

In the presence of faults, the input can be represented by

$$u = u^g + u^f \tag{2}$$

where $u^g \in \Re^q$ represents the fault-free input vector and $u^f \in \Re^q$ represents the actuator fault vector. It is assumed that u^g is available for computation but u^f and o are not. The magnitude of the noise is assumed to be significantly smaller then the magnitude of faults. Under the nominal fault-free condition, u^f is a zero vector. However, when an actuator fault occurs in the system, u^f will become non-zero. A schematic diagram of the overall system is given in Fig. 1.

MPM is represented by

$$f(x) = f^n(x) + f^u(x), \quad g(x) = g^n(x) + g^u(x) \tag{3}$$

where $f^n(x)$, $f^u(x)$, $g^n(x)$, and $g^u(x)$ represent the nominal and uncertain part of the mappings f and g, respectively. Combining (1)–(3), the overall system with faults is represented by

$$\dot{x} = f^n(x) + f^u(x) + (g^n(x) + g^u(x))(u^g + u^f) + d(x,u), \\ y = Cx + o \tag{4}$$

Simplifying (4)

$$\dot{x} = f^n(x) + g^n(x)u^g + e(x,u) + g^n(x)u^f, \quad y = Cx + o \tag{5}$$

where $e(x,u) = f^u(x) + g^u(x)(u^g + u^f) + d(x,u)$. The vector $e(x,u)$ is called an error vector, which contains both the uncertainty of the model and the disturbances. The following assumptions will be used in this paper in order to design the robust actuator fault detection method:

Assumption 1. The fault-free system is asymptotically stable. This is a general assumption in the FDI literature (Ding et al., 1999; Han et al., 2005).

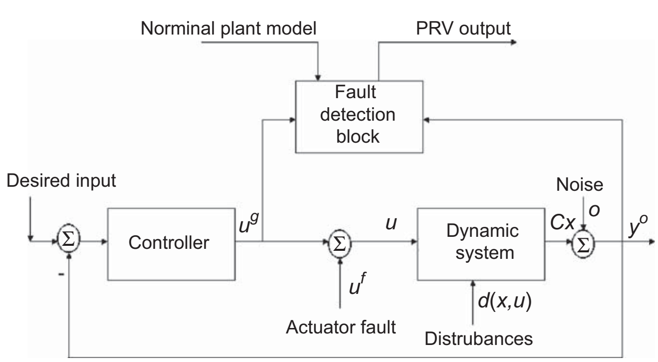

Fig. 1. Schematic diagram of the FDI system.

Assumption 2. The system in (1) is observable. This assumption is needed in order to guarantee the ability to find all the states from the system outputs and is also common in the literature (Gertler & Singer, 1990). Note that the observability assumption does not indicate that fault-free states can be found from faulty output measurements.

Assumption 3. The modeling uncertainty, denoted by $f^u(x)$ and $g^u(x)$ in (3), which are unknown nonlinear vector functions of x, is bounded. It is assumed that both the inputs and the disturbances are bounded, which is similar to the assumption made in Leuschen et al. (2005). Define fault-free error part, e^*, as $e^* = f^u(x) + g^u(x)u^g + d$. It is assumed that $\max\{||f^u(x)||, ||g^u(x)u^g||, ||d(x)||\} < ||F_o(x, u^g)||$ where $F_o(x, u^g)$ is a known bounded function. Now, $||e^*|| \leqslant ||f^u(x)|| + ||g^u(x)u^g|| + ||d||$. Thus e^* is bounded, e.g., $||e^*|| \leqslant L$, where $||\,||$ stands for the L_2 norm and $L = 3||F_o(x, u^g)||$.

Thus the problem in this paper becomes: design robust residuals for actuator faults for the nonlinear system given by (5). Robust means the residual will need to be sensitive to the faults but insensitive to the MPM and disturbances of the system, i.e., insensitive to error as much as possible.

2.1. RNLAR residual formulation

The major issue in the use of AR technique is how to deal with the presence of MPM and process disturbances, and their effect on the robustness of the resulting fault detection algorithm. In order to present the mathematical framework of robust residual generation several key matrices are defined first. Define the following matrices: a state matrix, Γ_s, an error matrix, G_s, and an input matrix, H_s

$$\Gamma_s = \begin{bmatrix} Cx \\ L_f Cx \\ L_{ff}Cx + \sum_{l=0}^{q}\sum_{j=0}^{q} L(j,l) \\ L_{fff}Cx + \sum_{k=0}^{q}\sum_{l=0}^{q}\sum_{j=0}^{q} L(j,l,k) \\ \cdots \end{bmatrix}_{m(s+1)\times 1} \tag{6}$$

$$G_s = \begin{bmatrix} 0 & 0 & 0 \\ C & 0 & 0 \\ C\left(\sum_{i=0}^{l} u_i^g \partial k_i/\partial x\right) & C & 0 \\ \Lambda & C\left(\sum_{i=0}^{l} u_i^g \partial k_i/\partial x\right) & C \end{bmatrix}_{m(s+1)\times ns} \tag{7}$$

$$H_s = \begin{bmatrix} 0 & 0 & 0 \\ Cg^n & 0 & 0 \\ C[(\partial f^n/\partial x)g^n + (\partial g^n/\partial x)f^n] & Cg^n & 0 \\ \Lambda_h & C[(\partial f^n/\partial x)g^n + (\partial g^n/\partial x)f^n] & Cg^n \end{bmatrix}_{m(s+1)\times qs} \tag{8}$$

where (Khalil, 1996; Slotine & Li, 1991)

$$L(j,l,m,\ldots) = (u_j u_l u_m \ldots)L_{\ldots k(m)k(l)k(j)}Cx.$$

$$u_0 = 1, \quad k(j) = \begin{Bmatrix} f, j = 0 \\ g_j, j \neq 0 \end{Bmatrix}$$

s describes the 'memory span' of the redundancy relation. The terms Λ and Λ_h contain higher order derivatives of the vector functions f^n and g_i^n.

Next, define a new group formation, O_{NDD}, which is based on the sensor reading and given control inputs. Take the derivative of

y for s times and stack them together in (9),

$$\begin{bmatrix} y \\ \dot{y} \\ \ddot{y} \\ \cdot \\ \cdot \\ y^s \end{bmatrix} = \begin{bmatrix} Cx + o \\ C\dot{x} + \dot{o} \\ C\ddot{x} + \ddot{o}. \\ \cdot \\ \cdot \\ Cx^s + o^s \end{bmatrix} \tag{9}$$

$$= \begin{bmatrix} Cx + y^f + o \\ C(f^n(x) + g^n(x)u^g + e(x,u) + g^n(x)u^f) + \dot{y}^f + \dot{o} \\ C\frac{d}{dt}(f^n(x) + g^n(x)u^g + e(x,u) + g^n(x)u^f) + \ddot{y}^f + \ddot{o} \\ \cdot \\ \cdot \end{bmatrix} \tag{10}$$

Define the stacked vectors, $y_s = [y \;\; \dot{y} \;\; \ddot{y} \;\; \dddot{y} \;\; \ldots]^T \in R^{m(s+1)}$, Similarly, define the input stack vector, u_s, the error stack vector, e_s, actuator fault stack vector, u_s^f, and the noise stacked vector, o_s as follows:

$$u_s = [u^g \;\; \dot{u}^g \;\; \ddot{u}^g \;\; \ldots]^T \in R^{qs}; \quad e_s = [e \;\; \dot{e} \;\; \ddot{e} \;\; \ldots]^T \in R^{ns}$$
$$o_s = [o \;\; \dot{o} \;\; \ddot{o} \;\; \ldots]^T \in R^{m(s+1)};$$
$$u_s^f = [u^f \;\; \dot{u}^f \;\; \ddot{u}^f \;\; \ldots]^T \in R^{qs}$$

Using the definitions of Γ_s, G_s, and H_s rearrange (10) as follows:

$$y_s = H_s u_s + \Gamma_s + y_s^f + G_s e_s + H_s u_s^f + o_s \tag{11}$$

Define O_{NDD} for RNLAR as follows:

$$O_{NDD} = y_s - H_s u_s \tag{12}$$

Eq. (12) together with the definition of O_{NDD} implies

$$O_{NDD} = \Gamma_s + y_s^f + G_s e_s + H_s u_s^f + o_s \tag{13}$$

Eqs. (12) and (13) will be used to derive the residuals for sensor and actuator faults. Note that in (12) y_s and u_s are outputs and inputs of the actual system. In (13) H_s, Γ_s and G_s are computed from the nominal system (i.e., the mathematical model of the plant).

2.2. Robust actuator fault detection

The objective is to design a residual vector that is less sensitive to the error vector and most sensitive to the actuator fault. The ideal outcome would be to design a residual vector that is only sensitive to the actuator fault and completely insensitive to the error vector. Let us investigate whether the ideal outcome can be achieved. Rearrange (13) to obtain

$$O_{NDD} = \Omega_s E_s + H_s u_s^f + o_s \tag{14}$$

where $\Omega_s = [\Gamma_s \;\; G_s]$ and $E_s = \begin{bmatrix} 1 \\ e_s \end{bmatrix}$.

Select a transformation matrix, c_s, from the space C_s defined by $C_s = \{c_s : c_s^T \Omega_s \equiv [0]\}$. Pre-multiplying both sides of (14) with c_s^T results:

$$R_a = c_s^T O_{NDD} = c_s^T H_s u_s^f + o_s \tag{15}$$

R_a is called the primary residual vector (PRV) for actuator fault. It appears that R_a is completely insensitive to the error vector. But note that, for a full rank C matrix c_s can only have its first m columns to be nonzero and the rest of the elements to be zero due to the block-triangular structure of the matrix G_s since $c_s^T G_s = 0$. Also note that the first m rows of H_s are zero. Hence $c_s^T H_s \equiv [0]$. Substituting this in (15) gives

$$R_a = c_s^T O_{NDD} = c_s^T o_s \tag{16}$$

Hence both the error vector and the fault contributing term $c_s^T H_s u_s^f$ are annihilated at the same time. This implies that the actuator residual is insensitive to not only the error vector but also to the fault. Therefore, no actuator fault can be detected if the error vector is completely removed when the outputs are non-redundant (i.e., C is a full row rank matrix).

Faced with the above problem, it can be concluded from (16) that complete elimination of the effect of the error vector from the PRV is not possible when $c_s^T \Omega_s \equiv [0] \Rightarrow c_s^T H_s = [0]$. This result is consistent with actuator fault detection results obtained for linear systems. In this case, a design methodology is presented for the PRV that makes it insensitive to the error vector but sensitive to the actuator faults as much as possible. Select a transformation vector, w_s, from the parity space W_s defined by $W_s = \{w_s : w_s^T \Gamma_s \equiv [0]\}$. The existence of w_s is guaranteed as Γ_s is a row vector. Pre-multiplying both sides of (13) with w_s^T results:

$$R_a = w_s^T (y_s - H_s u_s) = w_s^T (G_s e_s + H_s u_s^f + o_s) \tag{17}$$

It can be observed from (17) that the actuator residual, R_a, is sensitive to both the actuator faults and the uncertainty of the system. It is desirable that R_a should be highly sensitive to the actuator faults and mostly insensitive to the error terms in order to be able to detect actuator fault in the presence of error term. The above desired property can be translated mathematically into the statement, $||w_s^T G_s||$ is less than $||w_s^T H_s||$, where the coefficient of the error vector is $w_s^T G_s$ and the coefficient of the fault vector is $w_s^T H_s$. Both G_s and H_s are system dependent matrices. However, w_s can be chosen independently from the parity space to satisfy the above requirement. Hence the problem becomes, select a transformation vector w_s for the parity space in such a way that $||w_s^T G_s||$ is less than $||w_s^T H_s||$. In the literature this problem is discussed for linear systems. Both Isermann (1984) and Ding et al. (1999) frame this problem as a linear optimization problem and use the linearity property to determine w_s. For a nonlinear system, which is the case here, this translates into solving a nonlinear optimization problem where the functional structure of w_s is unknown. In other words, the functional form of each element of w_s is unknown (e.g., whether they are polynomial, exponential etc.) and cannot realistically be guessed without any other knowledge. This makes the nonlinear optimization problem very difficult to solve. In order to overcome this problem a novel method (Halder & Sarkar, 2007) is proposed for designing W_s for nonlinear systems.

Given the states $x \in \Re^n$ and inputs $u \in \Re^q$, consider an open set $U_r \in \Re^{n+q}$ such that the states and the inputs are restricted on U_r, i.e., $x_e = (x, u) \in U_r$. Define the performance function, $J_s(x,u) = w_s^T G_s G_s^T w_s / w_s^T H_s H_s^T w_s$.

The robust problem is formulated as follows: Find a w_s from the parity space such that $J \leqslant K(x_e)\, \forall x_e \in U_r$ where $K(x_e)$ predefined and $0 < K(x_e) < 1$. The choice of $K(x_e)$ will determine the sensitivity of actuator residual to the actuator fault and insensitivity to the error term. A small value of K will guarantee the sensitivity requirement of w_s. Here the subscripts from w_s is omitted and other terms for notational simplicity. Define $S_G = GG^T$, $S_H = HH^T$ and $R = S_G - KS_H$. Now using the newly defined notation, the above problem becomes:

Given Γ, G, and H, produce a vector function $w \in R^{m(s+1)}$ such that the following conditions are satisfied:

$$\begin{aligned} &1.\ w^T \Gamma \equiv [0], \quad w^T H \neq [0] \\ &2.\ w^T R w \leqslant 0, \quad \forall x_e \in U_r \end{aligned} \tag{18}$$

The following theorem is proposed in order to solve the above problem.

Theorem 1. *Part (i): Let $\mu^-(R)$ be the number of distinct, non-positive, eigenvalues of R. If, $\mu^-(R) \geqslant 2$ then $\exists w$ that satisfies both Conditions 1 and 2 in (18). Also, if λ_i are the non-positive eigenvalues of R and V_i are the corresponding eigenvectors, for $i \in \{1, 2, \ldots, n\}$,* *then* $w(x_e)\sum_{i=1}^{\mu^-} = \alpha_i(x_e)V_i(x_e)$ *satisfies Condition* 2. *For* $i\geqslant 2$, *it is possible to choose* $\alpha_i(x_e)$ *such that Condition* 1 *satisfies.*

Part (*ii*): *When* $\mu^-(R) = 1$ *then there exists* w *such that* $w = \alpha V$, *only if* $w^T\Gamma \equiv [0]$ *where* V *is the eigenvector corresponding to the non-positive eigenvalue of* R.

Part (*iii*): *If* $\mu^-(R) = 0$, *i.e., all the eigenvalues of R are positive, then there is no such w that satisfies both Conditions* 1 *and* 2. *The proof of the above theorem is given in the* Appendix A. *It was pointed out in* (Halder & Sarkar, 2007) *that an increase in the order of redundancy relation, s, increases the robustness.*

3. Robustness theorem

A theorem is formulated that shows that increasing the order of redundancy relations improves the system robustness in the sense of performance index J_s, where the subscript s represents the order of the redundancy relation. This is the main contribution of this paper. The theorem is stated as follows:

Theorem 2. *Given the states* $x \in \Re^n$ *and inputs* $u \in \Re^q$, *consider an open set* $U_r \in \Re^{n+q}$ *such that the states and the inputs are restricted in,* U_r *i.e.,* $X = (x, u) \in U_r$. *Let*

$$\alpha_s = \min_{w_s\in W_s}\max_{X\in U_r} J_s(X), \quad \alpha_{s+1} = \min_{w_{s+1}\in W_{s+1}}\max_{X\in U_r} J_{s+1}(X) \tag{19}$$

then $\alpha_s < \alpha_{s+1}$ *for all* $s > 0$.

The proof of the above theorem is given in the Appendix B.

Based on Theorem 2, a step-by-step procedure for optimal search of parity vector in the parity space for actuator fault detection can be constructed.

Step 1: Set the order of redundancy relation s and choose a desired value for K as defined before.

Step 2: Find the nature of eigenvalues of R based on the choice of K and s.

Step 3: If there are more then two distinct non-positive eigenvalues, then calculate w_s.

Step 4: If the above condition does not satisfy, then increase the value of s and go to Step 2.

4. Experimental results

A Unimate PUMA 560, as shown in Fig. 2, is used for experiment. PUMA is well-characterized industrial manipulator that has been utilized in numerous industrial and robotic research applications. PUMA is a three degree-of-freedom harmonic-drive manipulator with a three degree-of-freedom wrist attached at its endpoint. Recent researches on fault detection of robotic arm (Tino's, Terra, & Bergeman, 2007; Caccavale, Cilibrizzi, Pierri, & Villani, 2009) demonstrate the uncertain nature of robotic manipulator model.

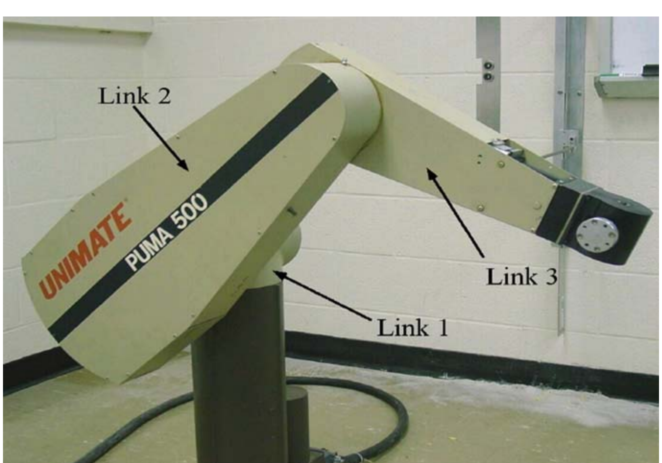

Fig. 2. Unimate PUMA 560 robot manipulator.

Armstrong, Khatib, and Burdick (1986) derived an explicit dynamic model of the PUMA 560 arm and measured the parameters necessary to implement model-based control. The equation of motion that is expressed in generalized coordinates of the PUMA arm, θ_1, θ_2 and θ_3, where θ_1 is the angle of rotation of the Link 1 about the vertical axis, θ_2 the angle measured from horizontal to Link 2, and θ_3 the angle measured from Link 2 to Link 3. They are represented in vector form by

$$\theta = [\theta_1\ \ \theta_2\ \ \theta_3]^T \tag{20}$$

In the absence of joint friction, the equation of motion for the robot manipulator is:

$$M(\theta)\ddot{\theta} + N(\theta)[\dot{\theta}\dot{\theta}] + P(\theta)[\dot{\theta}^2] + G(\theta) = T \tag{21}$$

where $M(\theta)$ represents the inertia matrix, $N(\theta)$ the matrix of Coriolis torques, $P(\theta)$ the matrix of centrifugal torques, $G(\theta)$ the vector of gravity torques, $[\theta\dot{\theta}]$ are notation for the vector of velocity products, $[\dot{\theta}^2]$ are vectors of squared velocities, and T the generalized joint force torques. The details of each term and the numeric parameters for the components of the model of the PUMA arm are given in (Armstrong et al., 1986).

Eq. (33) can be expressed in state space form as follows:

$$\dot{x} = f(x) + \sum_{i=1}^{3} g_i(x)u_i, \quad y = Cx = [\theta_1\ \ \theta_2\ \ \theta_3]^T \tag{22}$$

where $x = [\theta_1\ \ \theta_2\ \ \theta_3\ \ \dot{\theta}_1\ \ \dot{\theta}_2\ \ \dot{\theta}_3]^T$, $f(x) = [\dot{\theta}_1\ \ \dot{\theta}_2\ \ \dot{\theta}_3\ \ -M^{-1}(N[\theta\dot{\theta}]+ P[\dot{\theta}^2]) - G]^T$, $g(x) = [g_1\ \ g_2\ \ g_3] = \begin{bmatrix} 0_{3\times 3} \\ M^{-1} \end{bmatrix}_{6\times 3}$ $u = [u_1\ \ u_2\ \ u_3]^T \in \Re^3 = T$, C is a 3×6 output matrix and $y \in \Re^3$ is the fault-free actual process output. It is worth mentioning that (34) represents the nominal model of a PUMA 560. In the presence of the actuator fault and the MPM system is represented as in (5).

Using (34) fault detection residual is implemented on PUMA. In modeling the PUMA, the friction term is not considered. Also the parameter estimation for PUMA is not perfect. All these factors contribute to the MPM and the disturbances in the experiments. To calculate the residuals as in (17) the nominal model simulated in parallel with the PUMA. The nominal model is used as in (34) to calculate the terms, G_s, H_s, and Γ_s while y_s and u_s are obtained directly from the experimental data. G_s, H_s, and Γ_s are formulated based on (6), (7), and (8). One concern about the implementation is to obtain the successive derivatives of input and output data in the presence of sensor noise. Numerical differentiation was performed using the derivative block, which uses forward differentiation technique, available in Matlab. Numerical differentiation of noisy sensor signal is well known to be ill posed in the sense that a small noise in measurement data can induce a large error in the approximate derivative. Although reasonable results was obtained in the experiments using the standard numerical differentiation block provided by Matlab, one can use various low-pass-filters and regularization methods such as Savitzky–Golay smoothing filters to reduce the effect of noise in differentiation if needed. Finally, generating the RNLAR residuals for PUMA require mathematical calculation of various terms. The Matlab symbolic toolbox and Mathematica was used to generate all the terms and combine them appropriately to create the RNLAR tests.

4.1. Results

The experiments are designed to detect actuator faults using different residuals: one RNLAR residual with $s=2$ and one RNLAR residual with $s=3$. In the experiment, the PUMA was asked to track a circular trajectory with radius of 0.10 meters in the x–y plane. The desired trajectory along with the actual trajectory under partial second actuator fault is given later in Fig. 7. The trajectory for x direction starts after 5 s. The endpoint of PUMA is controlled by a PID controller with the following PID gains: $p=400$, $I=5$, and $D=15$. It is worth to note that the residuals are independent of the choice of controller. Residual depends on the fault.

Various actuator faults are discussed in Luca and Mattone (2003). Two common actuator faults are used for the experiments. First one is a partial actuator fault where one actuator generates only a part of the desired torque. This type of fault represents degradation in the actuator system (e.g., friction due to jamming, problems in transmission etc.). The second actuator fault is a constant torque output. This may occur due to constant polarization of the actuator, called actuator bias.

In the experimental set-up the partial actuator faults are introduced in the second joint where the joint torque was reduced to 80% after 11 s of operation. Faults are considered detected if the magnitudes of the residuals cross a threshold value. A threshold is designed based on the statistical theory of the Z-test to reduce the false positives. The residual signal is compared against this threshold to detect faults. The mean, η and the variance, σ of the residual under no fault condition are calculated. From the statistical theory of Z-test the confidence limits of the mean that represent a confidence of $(1-\alpha)$ is $P\{\bar{\eta}-z\sigma<\eta<\bar{\eta}+z\sigma\}=1-\alpha$ where α is the confidence level, and z is the coefficient related to the confidence level. In this experimental work, a 99% confidence level is selected, i.e., $\alpha=0.01$. The corresponding z coefficient is 2.67. Hence the threshold is chosen as $\delta_{th}=\eta\pm 2.67\sigma$. The output of RNLAR residual with $s=2$ and 3 without any fault in the system is shown in Figs. 3 and 4 to demonstrate the effect of the MPM and process disturbance on the residuals. RNLAR residuals with $s=2$ and 3 under partial second actuator fault are shown is Figs. 5 and 6, respectively. In Fig. 5, the threshold value is shown in red dotted line. The peak value of the residual output for $s=2$ is 33.67, which is more than the threshold value. Observe that the residual is obtained using differentiation of the system outputs hence it is detecting the sudden change in the system. That is why the peak appears at the fault. A zoomed version of Fig. 5 around the peak is given in Fig. 5. It is observed in Fig. 5 that the peak remains for more than one sample point which is required for particle fault detection purpose. The peak value for RNLAR residual with $s=3$ is 3612, which is about 100 times more than the threshold value. Between the two residuals, the residual with $s=3$ is more sensitive to fault in the presence of

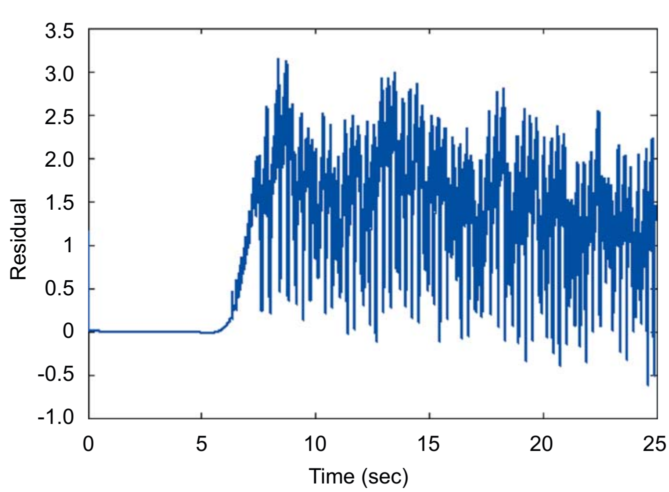

Fig. 3. RNLAR residual for $s=2$ without any fault.

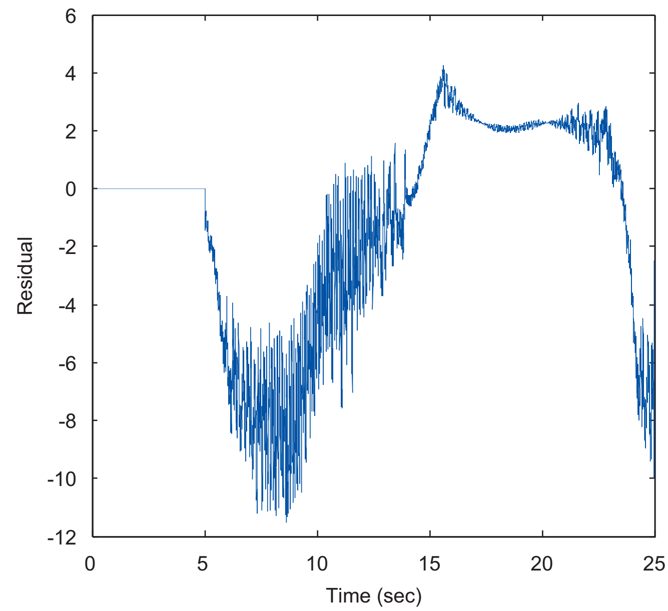

Fig. 4. RNLAR residual for $s=3$ without any fault.

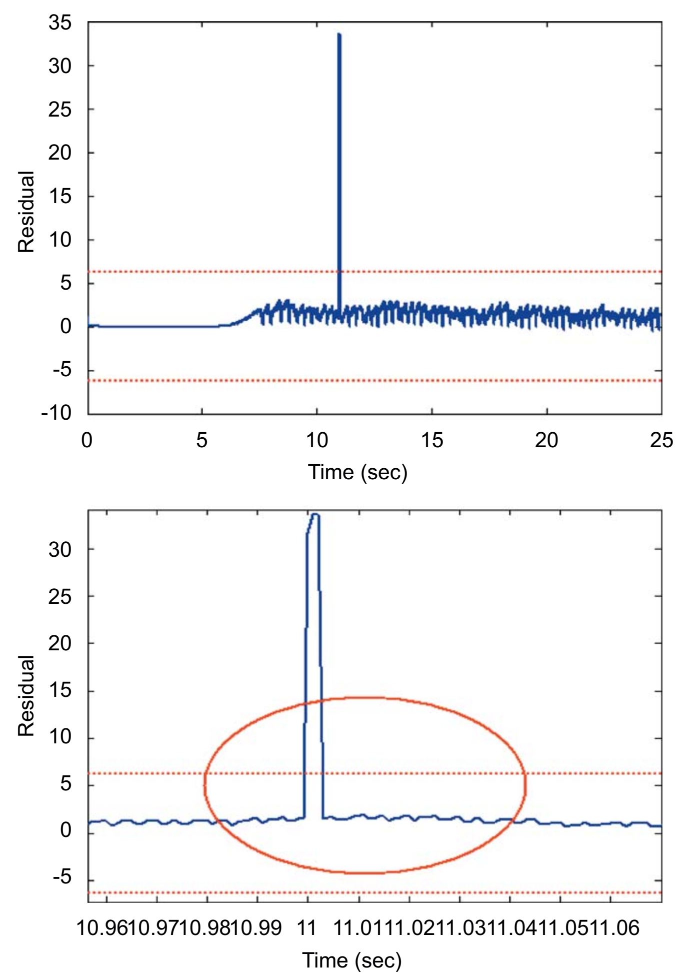

Fig. 5. RNLAR residual for $s=2$ under partial second actuator fault.

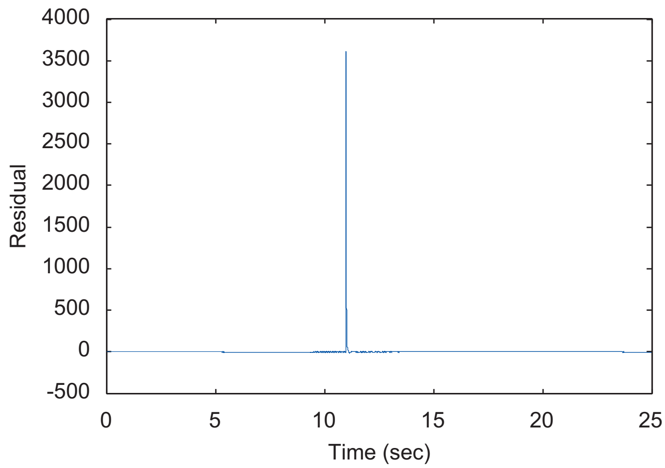

Fig. 6. RNLAR residual for $s = 3$ under partial second actuator fault.

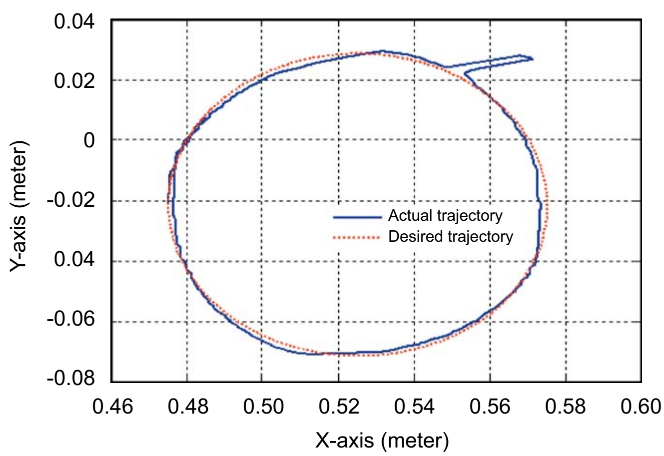

Fig. 7. Desired and actual trajectory under second actuator fault.

identical MPM and disturbance and hence is more robust. In Fig. 7 the desired and actual trajectory of robot tip position is given.

Next, another RNLAR residual is designed to detect the fault in third actuator. A constant $\tau_3 = 0.1$ is introduced to the third actuator at $t = 11$ s. The outputs of RNLAR residual with $s = 2$ and 3 without any fault are shown in Figs. 8 and 9. RNLAR residuals with $s = 2$ and 3 under partial second actuator are shown is Figs. 10 and 11, respectively. The peak value of the residual output for $s = 2$ is 42.19, which is more than the threshold value. Hence the fault is considered detected. The peak value for RNLAR residual with $s = 3$ is 274.94. Thus once again notice that increasing the redundancy order increases the robustness in fault detection.

5. Conclusion

In this paper, the relation between order of redundancy relation and robustness of the system was studied. The RNLAR residuals generation procedure is presented for multivariable input-affine nonlinear dynamic systems. The main contribution of this paper is to formulate and prove the theorem that increasing the order of redundancy relation improves the system robustness. The proposed theorem is an extension of the similar results obtained in linear systems. The presented method works in the presence of modeling uncertainty and disturbances as long as these are bounded and well behaved under differentiation. It should be noted that while increasing the order of redundancy

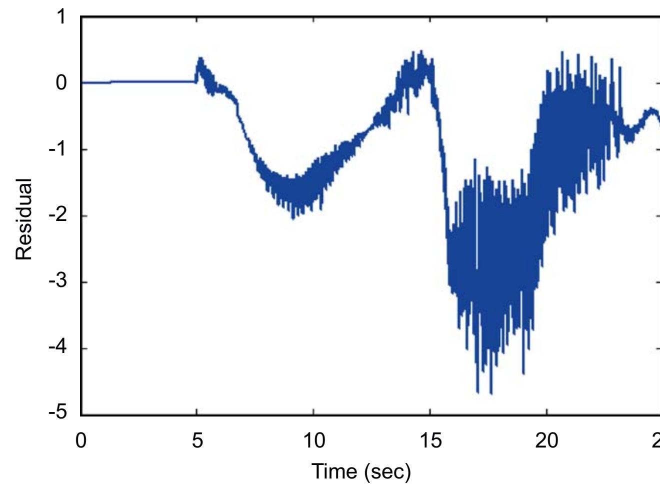

Fig. 8. RNLAR residual for $s = 2$ without any fault.

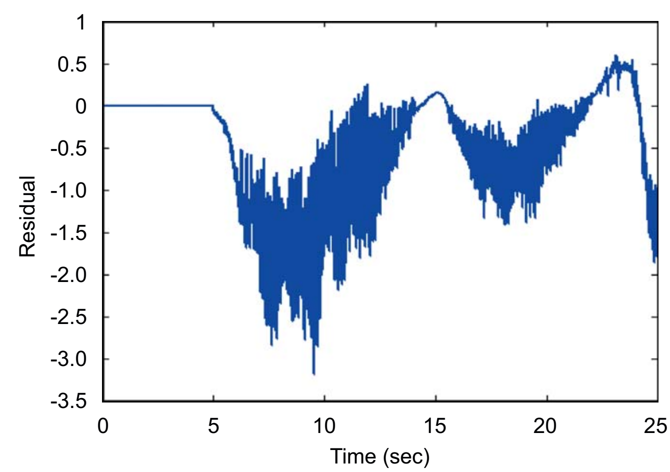

Fig. 9. RNLAR residual for $s = 3$ without any fault.

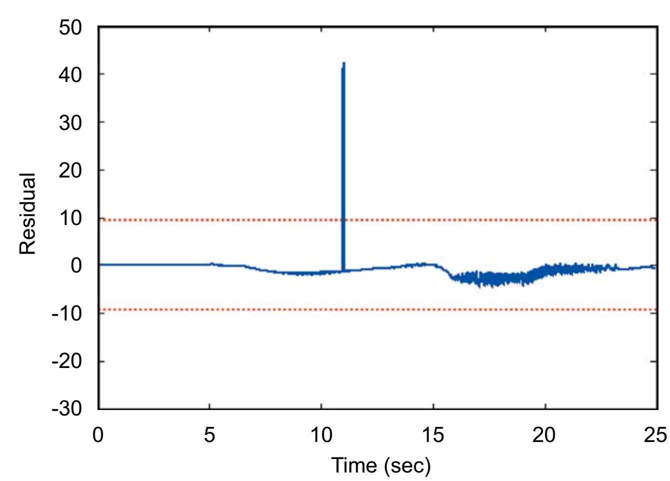

Fig. 10. RNLAR residual for $s = 2$ under bias third actuator fault.

mathematically improves the system robustness, there could be practical limitations on the order of differentiation due to computational issues. In particular, for noisy signal increasing the order of differentiation may amplify the effect of noise to offset the robustness gain. It is thus practical to design the actuator fault residual with the lowest order of redundancy that is

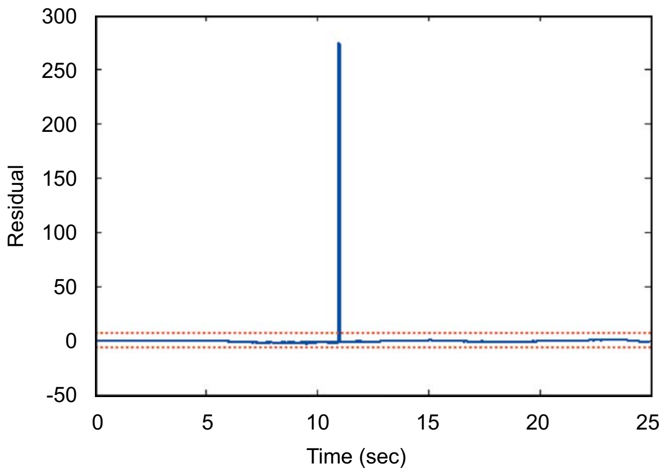

Fig. 11. RNLAR residual for $s=3$ under bias third actuator fault.

adequate to detect faults. For mechanical systems that are characterized by second order dynamics, $s=2$ or 3 may be sufficient for most cases. It may pose a potential problem in certain applications if much higher value of s is required to detect fault due to computational cost and noise amplification.

Based on the theorem, an algorithm is proposed to determine the optimal redundancy relation order starting from the lowest possible value of s. The usefulness of the theorem was experimentally demonstrated on a PUMA 560 robotic arm. A comparative experimental study has been presented to demonstrate the effect of robust residuals.

Acknowledgments

We gratefully acknowledge the ARO Grant DAAD19-02-1-0160 and ONR Grant N00014-03-1-0052 and N00014-06-1-0146 that partially supported this work.

Appendix A

Proof. *Part* (*i*): The eigenvalues of R, $\lambda_i(x_e)$, is a function of x_e. $\lambda_i(x_e)$ is positive if $\lambda_i(x_e)>0\ \forall x_e \in U_r$, and negative if $\lambda_i(x_e)<0\ \forall x_e \in U_r$.

Let us consider the case where $\mu^-(R)\geqslant 2$. It is needed to prove that $w=\sum_{i=1}^{\mu^-}\alpha_i V_i$ satisfies Condition 2. Without loss of generality consider $i=2$, let λ_1 and λ_2 be the non-positive eigenvalues and V_1 and V_2 be the corresponding eigenvectors. Then $w(x_e)=\alpha_1(x_e)V_1(x_e)+\alpha_2(x_e)V_2(x_e)$, where $\alpha_1(x_e)$ and $\alpha_2(x_e)$ are the chosen coefficients.

Observe that R is a symmetric matrix. To see this,

$$S_G^T=(GG^T)^T=(G^T)^TG^T=GG^T=S_G$$

This implies S_G is symmetric. For similar reason S_H is also symmetric. R is the linear combination of S_G and S_H, hence R is also symmetric.

Now,

$$\begin{aligned} w^TRw &= (\alpha_1V_1^T+\alpha_2V_2^T)R(\alpha_1V_1+\alpha_2V_2)\\ &= (\alpha_1V_1^T+\alpha_2V_2^T)(\alpha_1RV_1+\alpha_2RV_2)\\ &= (\alpha_1V_1^T+\alpha_2V_2^T)(\alpha_1\lambda_1IV_1+\alpha_2\lambda_2IV_2)\\ &= \alpha_1^2\lambda_1V_1^TV_1+\alpha_2^2\lambda_2V_2^TV_2+\alpha_1\alpha_2(\lambda_2V_1^TV_2+\lambda_1V_2^TV_1)\end{aligned}$$

where I is the identity matrix and V_1^T and V_2^T represent the transpose of V_1 and V_2, respectively. Since R is a symmetric matrix, it implies $V_2^TV_1=0$ and $V_1^TV_2=0$. Hence $w^TRw=\lambda_1\alpha_1^2V_1^TV_1+\lambda_2\alpha_2^2V_2^TV_2$. But $\alpha_1^2V_1^TV_1$ and $\alpha_2^2V_2^TV_2$ are always positive, hence $w^TRw\leqslant 0$ for $\lambda_1, \lambda_2\leqslant 0$.

This proves the first part of Part (*i*).

The second part of the claim, i.e., for $i\geqslant 2$ it is possible to choose $\alpha_i(x_e)$ such that the Condition 1 is satisfied. This is obvious because there is only one constraint and more than one variable. This completes the proof of first claim.

Part (*iii*): Lets assume that $\exists$ nonzero w that satisfies Conditions 1 and 2 when $\mu^-(R)=0$. More specifically w satisfies $w^TRw\leqslant 0\ \forall x$. R is a symmetric matrix with all positive eigenvalues. That implies R is a positive definite matrix, which then means $X^TRX>0\ \forall X\neq 0$, where X is an arbitrary nonzero vector. This is a contradiction. This completes the proof.

Part (*ii*): This is a direct consequence of the other two claims. This completes the proof of the theorem. □

Appendix B

Proof. For a given s, let $X_{cs}\in U_r$ and$w_{cs}\in W_s$ be the optimal choice such that

$$\alpha_s=\frac{w_{cs}^TG_sG_s^Tw_{cs}}{w_{cs}^TH_sH_s^Tw_{cs}} \tag{23}$$

To prove the inequality $\alpha_s>\alpha_{s+1}$, it is sufficient to show that there exists a vector $w_{s+1}\in W_{s+1}$ such that

$$\frac{w_{s+1}^TG_{s+1}G_{s+1}^Tw_{s+1}}{w_{s+1}^TH_{s+1}H_{s+1}^Tw_{s+1}}<\alpha_s \tag{24}$$

for $X_{cs}\in U_r$. That implies

$$w_{s+1}^T(G_{s+1}G_{s+1}^T-\alpha_sH_{s+1}H_{s+1}^T)w_{s+1}<0 \tag{25}$$

Express G_{s+1} and H_{s+1} in terms of G_s and H_s, respectively. G_s is a $m(s+1)\times ns$ matrix while the dimension of G_{s+1} is $m(s+2)\times n(s+1)$. G_{s+1} has m more rows and n more columns than that of G_s, which carries the information of $(s+1)$th order differentiation of the output equation. Hence, express G_{s+1} as follows:

$$G_{s+1}=\begin{bmatrix} \underset{m(s+1)\times ns}{G_s} & \underset{m(s+1)\times n}{0}\\ \underset{m\times ns}{\Lambda_g} & \underset{m\times n}{\Delta_g}\end{bmatrix}_{m(s+2)\times n(s+1)} \tag{26}$$

where Λ_g and Δ_g contain the $(s+1)$th order differentiation of the output equation that is used with the error term. In a similar fashion, H_{s+1} can be expressed in terms of H_s as follows:

$$H_{s+1}=\begin{bmatrix} \underset{m(s+1)\times qs}{H_s} & \underset{m(s+1)\times q}{0}\\ \underset{m\times qs}{\Lambda_h} & \underset{m\times q}{\Delta_h}\end{bmatrix}_{m(s+2)\times q(s+1)} \tag{27}$$

where Λ_h and Δ_h contain the $(s+1)$th order differentiation. Substituting (23) and (24) into (22) gives

$$w_{s+1}^T\left(\begin{array}{l}\begin{bmatrix} \underset{m(s+1)\times ns}{G_s} & \underset{m(s+1)\times n}{0}\\ \underset{m\times ns}{\Lambda_g} & \underset{m\times n}{\Delta_g}\end{bmatrix}\begin{bmatrix} \underset{m(s+1)\times ns}{G_s} & \underset{m(s+1)\times n}{0}\\ \underset{m\times ns}{\Lambda_g} & \underset{m\times n}{\Delta_g}\end{bmatrix}^T\\ -\alpha_s\begin{bmatrix} \underset{m(s+1)\times qs}{H_s} & \underset{m(s+1)\times q}{0}\\ \underset{m\times qs}{\Lambda_h} & \underset{m\times q}{\Delta_h}\end{bmatrix}\begin{bmatrix} \underset{m(s+1)\times qs}{H_s} & \underset{m(s+1)\times q}{0}\\ \underset{m\times qs}{\Lambda_h} & \underset{m\times q}{\Delta_h}\end{bmatrix}^T\end{array}\right)w_{s+1}$$

$$=w_{s+1}^T\left(\begin{bmatrix} \underset{m(s+1)\times m(s+1)}{G_sG_s^T} & \underset{m(s+1)\times m}{G_s\Lambda_g^T}\\ \underset{m\times m(s+1)}{\Lambda_gG_s^T} & \underset{m\times m}{\Lambda_g\Lambda_g^T+\Delta_g\Delta_g^T}\end{bmatrix}\right.$$

$$- \alpha_s \begin{bmatrix} \underset{m(s+1)\times m(s+1)}{H_s H_s^T} & \underset{m(s+1)\times m}{H_s \Lambda_h^T} \\ \underset{m\times m(s+1)}{\Lambda_h H_s^T} & \underset{m\times m}{\Lambda_h \Lambda_h^T + \Delta_h \Delta_h^T} \end{bmatrix} \Bigg) w_{s+1}$$

$$= w_{s+1}^T \begin{pmatrix} G_s G_s^T - \alpha_s H_s H_s^T & G_s \Lambda_g^T - \alpha_s H_s \Lambda_h^T \\ \Lambda_g G_s^T - \alpha_s \Lambda_h H_s^T & \Lambda_g \Lambda_g^T + \Delta_g \Delta_g^T \\ & -\alpha_s(\Lambda_h \Lambda_h^T + \Delta_h \Delta_h^T) \end{pmatrix} w_{s+1} \quad (28)$$

Construct w_{s+1} as follows

$$w_{s+1} = \begin{bmatrix} w_{cs} \\ \gamma w_e \end{bmatrix}_{m(s+2)\times 1}$$

where γ is a scalar constant and w_e is a $m \times 1$ vector. Construct w_e such a way that (22) is satisfied and $w_{s+1}^T \Gamma_{s+1} = 0$. Substituting the above choice of w_{s+1} in (25) gives (26).

$$[w_{cs}^T \;\; \gamma w_e^T] \begin{pmatrix} G_s G_s^T - \alpha_s H_s H_s^T & G_s \Lambda_g^T - \alpha_s H_s \Lambda_h^T \\ \Lambda_g G_s^T - \alpha_s \Lambda_h H_s^T & \Lambda_g \Lambda_g^T + \Delta_g \Delta_g^T - \alpha_s(\Lambda_h \Lambda_h^T + \Delta_h \Delta_h^T) \end{pmatrix} \times \begin{bmatrix} w_{cs} \\ \gamma w_e \end{bmatrix} \quad (29)$$

Let's define $A = G_s \Lambda_g^T - \alpha_s H_s \Lambda_h^T$, $B = \Lambda_g G_s^T - \alpha_s \Lambda_h H_s^T$ and $C = \Lambda_g \Lambda_g^T + \Delta_g \Delta_g^T - \alpha_s(\Lambda_h \Lambda_h^T + \Delta_h \Delta_h^T)$. Substituting the above definition

$$[w_{cs}^T \;\; \gamma w_e^T] \begin{pmatrix} G_s G_s^T - \alpha_s H_s H_s^T & A \\ B & C \end{pmatrix} \begin{bmatrix} w_{cs} \\ \gamma w_e \end{bmatrix} \quad (30)$$

$$= w_{cs}^T (G_s G_s^T - \alpha_s H_s H_s^T) w_{cs} + \gamma w_e^T B w_{cs} + w_{cs}^T A \gamma w_e + \gamma w_e^T C \gamma w_e \quad (31)$$

$$= \gamma w_e^T B w_{cs} + w_{cs}^T A \gamma w_e + \gamma w_e^T C \gamma w_e \quad (32)$$

because $w_{cs}^T (G_s G_s^T - \alpha_s H_s H_s^T) w_{cs} = 0$. Now, for $m \geqslant 2$, w_e can be selected such a way that $\gamma w_e^T B w_{cs} + w_{cs}^T A \gamma w_e + \gamma w_e^T C \gamma w_e < 0$ and $w_{s+1}^T \Gamma_{s+1} = 0$ as two constraints can be satisfied with two free variables. With the above selection of w_e, finally

$$\begin{aligned} &\gamma w_e^T B w_{cs} + w_{cs}^T A \gamma w_e + \gamma w_e^T C \gamma w_e < 0 \\ &\Rightarrow w_{s+1}^T (G_{s+1} G_{s+1}^T - \alpha_s H_{s+1} H_{s+1}^T) w_{s+1} < 0 \\ &\Rightarrow \frac{w_{s+1}^T G_{s+1} G_{s+1}^T w_{s+1}}{w_{s+1}^T H_{s+1} H_{s+1}^T w_{s+1}} < \alpha_s \end{aligned} \quad (33)$$

For $m = 1$, w_e is a scalar quantity. Hence write (30) as

$$\gamma w_e B w_{cs} + \gamma w_e w_{cs}^T A + \gamma^2 w_e w_e C \quad (34)$$

Choose the constant γ such a way that $\gamma w_e B w_{cs} + \gamma w_e w_{cs}^T A + \gamma^2 w_e w_e C < 0$ when both A and B are non-zero, which is the case here. Then select w_e such a way that $w_{s+1}^T \Gamma_{s+1} = 0$. This concludes the proof of the Theorem. □

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