On the Observability of Pressure in a Pneumatic Servo Actuator

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Abstract
Pneumatic servo actuators are characterized by highly nonlinear dynamics from spool valve command to cylinder pressure, and as such are well suited to the use of nonlinear control methods requiring measurement of the full state, such as sliding mode control. This paper investigates the possibility of eliminating the pressure measurement required for nonlinear control in lieu of a nonlinear pressure observer, based on measurement of the output motion. A nonlinear observability analysis is conducted that demonstrates that such a system loses local observability in several regions of the state space, thus rendering the system unobservable.

1 Introduction

A typical pneumatic servo system, which consists primarily of a proportionally controllable 4-way spool valve and a pneumatic cylinder, is depicted in Fig. 1. In this system, the position of the valve spool controls the flow into and out of each side of the cylinder, which in turn results in a pressure differential across the piston and thus imposes a force on the load. The (measurable) state variables of such a system are typically the pressures in each cylinder chamber, which are required to characterize the energy storage due to the compressibility of air, the velocity of the load, which is required to characterize the energy stored by the load inertia, and the position of the load, which is the desired output. The behavior of this type of system is characterized by highly nonlinear dynamics between the valve spool position input and the piston rod position output. These nonlinear dynamics arise principally from the compressibility of air, which give rise to two nonlinear components of the system dynamics. The first is the nonlinear relationship that describes the compliance of an ideal gas in each side of the cylinder, and the second is the effective mass flow rate saturation that occurs when the compressible fluid flow through the valve transitions from the subsonic to the sonic flow regime. Specifically, in the subsonic flow regime, the mass flow rate through the valve is a function of the pressure drop across the valve. Once the flow through the valve becomes sonic, however, the conditions downstream of the valve no longer affect the upstream flow, since pressure disturbances travel at the speed of sound, and thus the changes in the downstream pressure cannot travel upstream quickly enough to affect the upstream flow. If the airflow through the valve is assumed isentropic (i.e., adiabatic and frictionless), then the transition from subsonic to sonic flow will occur at a pressure ratio of upstream to downstream absolute pressure of approximately two. Thus, when the (absolute) upstream pressure is at least twice the (absolute) downstream, the flow through the valve will become sonic and the mass flow rate will depend only upon the upstream pressure rather than the pressure drop across the valve. This condition of dependence on the upstream pressure only is called choked flow. Note that for frictional flow (as opposed to isentropic), choked flow will occur at a somewhat higher ratio of upstream to downstream pressure (i.e., somewhat more than two). Note that the presence of choked flow can be avoided by limiting the system supply pressure to one atmosphere gage. In this manner, the ratio of upstream to downstream pressure is limited to a maximum of two, and thus the choked flow condition will never occur (assuming isentropic flow). Such a low supply pressure (101 kPag or 14.7 psig), however, is almost never utilized in the control of pneumatic servo systems, since such a system would suffer from extremely low output impedance and severe power limitations. Therefore, for most practical purposes, the transition between choked (sonic) and unchoked (subsonic) flow in a pneumatic servo system is unavoidable. As such, pressure sensors are commonly employed in nonlinear model-based controllers of pneumatic servo systems in order to detect and compensate for the shift in dynamic behavior that occurs in the transition between choked and unchoked flow through the valve. Two pressure sensors are generally required per actuator, and the combined cost of these sensors often constitutes a significant portion of the cost of the pneumatic servo actuator package.

Since, as previously mentioned, the cylinder pressures are states of the pneumatic system, the possibility exists of eliminating the pressure sensors in lieu of a nonlinear observer. Specifically, if the motion output (i.e., position and velocity) of the piston can be measured, then a nonlinear observer might enable the reconstruction of cylinder pressures, and thus eliminate the need for pressure sensors. Despite the prior publication of control methodologies requiring full state measurement (see, for example, [1-7]), no prior published work explicitly treats the observability of cylinder pressure in such systems. This paper investigates the possibility of constructing a nonlinear observer to reconstruct the cylinder pressure states.

2 Model of a Pneumatic Servo Actuator

Models of a standard pneumatic servo actuator have been derived by Shearer [8-10], Burrows [11], McCloy and Martin [12], and Richer and Hurmuzlu [13], among others. In order to provide a basis for the observability analysis, the salient features of the standard dynamic model are developed briefly here. The load dynamics of the system shown in Fig. 1 can be written as:

$$M\ddot{z} + B\dot{z} = P_a A_a - P_b A_b - P_{atm} A_r \tag{1}$$

where z is the displacement of the load, M is the combined mass of the load, piston, and rod assembly, B is the viscous friction coefficient, P_a and P_b are the absolute pressures in chambers a and b, respectively, P_{atm} is atmospheric pressure, A_a and A_b are the effective areas of each side of the piston, and A_r is the cross-sectional area of the piston rod.

The dynamics of the chamber pressures P_a and P_b can be derived by utilizing the first law of thermodynamics and assuming no heat loss occurs in the cylinder (i.e., the heat loss is small relative to the work and enthalpy terms). As such, the change in internal energy in each chamber results solely from the addition or removal of enthalpy and work:

$$\dot{U}_{(a,b)} = \dot{H}_{(a,b)} - \dot{W}_{(a,b)} \tag{2}$$

where $U_{(a,b)}$ is the internal energy of the air in either chamber *a* or *b*, $H_{(a,b)}$ is the enthalpy added to either chamber *a* or *b*, and $W_{(a,b)}$ is the work done by chamber *a* or *b*. From the definition of internal energy, its time rate of change can be expressed as

$$\dot{U}_{(a,b)} = \frac{d}{dt}\left(m_{(a,b)}c_v(T_{(a,b)} - T_r)\right) = \frac{d}{dt}\left(m_{(a,b)}c_v T_{(a,b)}\right) = \frac{d}{dt}\left(\rho_{(a,b)}V_{(a,b)}c_v T_{(a,b)}\right) \tag{3}$$

where $m_{(a,b)}$, $T_{(a,b)}$, $V_{(a,b)}$, and $\rho_{(a,b)}$ are the mass, temperature, volume, and density of air in chamber *a* and *b*, respectively, T_r is the reference temperature used as the datum for measuring internal energy, and c_v is the specific heat of air at constant volume. If air is assumed to be an ideal gas, then

$$\rho_{(a,b)}T_{(a,b)} = \frac{P_{(a,b)}}{R} = \frac{P_{(a,b)}}{c_p - c_v} \tag{4}$$

and Eq. (3) can be rewritten as

$$\dot{U}_{(a,b)} = \frac{d}{dt}\left(\frac{c_v P_{(a,b)}V_{(a,b)}}{c_p - c_v}\right) \tag{5}$$

The time rate of change of enthalpy and work can be expressed in thermodynamic quantities as

$$\dot{H}_{(a,b)} = \dot{m}_{in(a,b)}c_p T_{in(a,b)} - \dot{m}_{out(a,b)}c_p T_{(a,b)} \tag{6}$$

$$\dot{W}_{(a,b)} = P_{(a,b)}\dot{V}_{(a,b)} \tag{7}$$

where it is assumed in the derivation of Eq. (6) that the enthalpy term associated with the time rate of change of temperature is small relative to that associated with the mass flow rate. Substituting Eqs. (5-7) into Eq. (2) yields

$$\dot{P}_{(a,b)}V_{(a,b)} + \gamma P_{(a,b)}\dot{V}_{(a,b)} = R\gamma T_{in(a,b)}\dot{m}_{in(a,b)} - R\gamma T_{(a,b)}\dot{m}_{out(a,b)} \tag{8}$$

where $\gamma = \dfrac{c_p}{c_v}$. Note that since each chamber has only a single port, it cannot be simultaneously charged and discharged. As such, the pressure dynamics in each chamber will assume either the charging form

$$\dot{P}_{(a,b)}V_{(a,b)} + \gamma P_{(a,b)}\dot{V}_{(a,b)} = R\gamma T_{in(a,b)}\dot{m}_{in(a,b)} \tag{9}$$

or the discharging form

$$\dot{P}_{(a,b)}V_{(a,b)} + \gamma P_{(a,b)}\dot{V}_{(a,b)} = -R\gamma T_{(a,b)}\dot{m}_{out(a,b)} \tag{10}$$

These pressure dynamics are driven by the mass flow rate term, which in turn is directly influenced by the commanded area of each valve. The relationship between the valve area and the mass flow rate of air is derived by assuming the flow through the valve to be an ideal gas undergoing an isentropic process, which leads to the commonly accepted mass flow rate expressions for a converging nozzle:

$$\dot{m}_{in/out(a,b)} = \begin{cases} \dfrac{C_1 C_d A_v P_u}{\sqrt{T_u}} & \text{if } \dfrac{P_d}{P_u} \le C_r \text{ (choked)} \\[2ex] \dfrac{C_2 C_d A_v P_u}{\sqrt{T_u}}\left(\dfrac{P_d}{P_u}\right)^{(1/\gamma)}\sqrt{1-\left(\dfrac{P_d}{P_u}\right)^{(\gamma-1)/\gamma}} & \text{otherwise (unchoked)} \end{cases} \tag{11}$$

where C_d is the discharge coefficient of the valve, P_u and P_d are the upstream and downstream pressures, respectively, T_u is the upstream air temperature, C_r is the pressure ratio that divides the flow regimes into unchoked and choked flow (approximately 0.5 for air), and C_1 and C_2 are constants defined as:

$$C_1 = \sqrt{\frac{\gamma}{R}\left(\frac{2}{\gamma+1}\right)^{(\gamma+1)/(\gamma-1)}} \tag{12}$$

and

$$C_2 = \sqrt{\frac{2\gamma}{R(\gamma-1)}} \tag{13}$$

The state equations for this system are thus given by:

$$\dot{x} = f(x,u) \tag{14}$$

where the state vector is defined as $x^T = [z \quad \dot{z} \quad P_a \quad P_b]$, the input is defined as $u = A_{va} = -A_{vb} = A_v$ (where a positive valve area indicates a connection to the supply pressure and negative indicates connection to exhaust), and the state functions are given by:

$$f = \begin{bmatrix} \dot{z} \\ \frac{1}{M}\left(-B\dot{z} + P_a A_a - P_b A_b - P_{atm} A_r\right) \\ f_3 \\ f_4 \end{bmatrix} \tag{15}$$

$$f_{(3,4)} = \begin{cases} -\frac{\gamma}{V_{(a,b)}}\left(P_{(a,b)}\dot{V}_{(a,b)} - RT_{in(a,b)}\dot{m}_{in(a,b)}\right) & \text{if } A_{v(a,b)} \geq 0 \text{ (charging)} \\ -\frac{\gamma}{V_{(a,b)}}\left(P_{(a,b)}\dot{V}_{(a,b)} + RT_{(a,b)}\dot{m}_{out(a,b)}\right) & \text{otherwise (discharging)} \end{cases} \tag{16}$$

where the mass flow rate is given by Eqs. (11-13). Note that the volume and rate of change of volume in Eq. (16) are algebraically related to the displacement and velocity of the piston, and therefore do not give rise to independent states. Note also that the system dynamics switch between eight distinct dynamic regimes. Specifically, four dynamic regimes exist for the combination of charging chamber *a* unchoked or choked while discharging chamber *b* unchoked or choked, and four dynamic regimes for charging chamber *b* unchoked or choked while discharging chamber *a* unchoked or choked.

3 Nonlinear Observability of Pressure

Nonlinear model-based controllers require measurement of the full state, which requires measurement of the load position and velocity in addition to the pressure in each chamber. Since the cylinder pressures are states of the dynamic system, the possibility exists of eliminating the pressure sensors through the use of a nonlinear observer. In order to reconstruct the pressure states from the motion states, the system must be observable. The following paragraph outlines a (previously published) necessary condition for the observability of nonlinear systems [14].

Consider the nonlinear system given by

$$\Sigma : \begin{cases} \dot{x} = f(x,u) \\ y = h(x) \end{cases} \tag{17}$$

where $x \in R^n$ is the state vector that is an element of an n-dimensional manifold N (i.e., $x \in N$), $u \in R^p$ is the input vector, and $y \in R^m$ is the measurable output of the system Σ. This system is said to be observable at x_0 (or alternatively at time t_0) if the state vector $x(t_0)$ can be determined from the observation of $y(t)$ over a finite time interval, $t_0 \le t \le t_1$. Hermann and Krener in 1977 proposed a rank condition test for what they termed "local weak observability" of a nonlinear system [14]. If the system Σ is locally weakly observable, then "one can instantaneously distinguish each point from its neighbors." It should be noted that local observability is a stronger condition than global observability. The former indicates that only the local state space is required to distinguish between states, while the latter may require the system "to travel a considerable distance or for a long time to distinguish between points of N." The notion of "weak" observability is a weakening of the local observability condition that requires that a given state need only be distinguished from local states rather than from the entire manifold N. Note that local weak observability is a necessary condition for local observability.

A necessary condition for local weak observability is stated as *Theorem 3.11* in [14] as follows: *If Σ is locally weakly observable, then the observability rank condition is satisfied generically.* In other words, if the system fails to satisfy the proposed rank condition, then it will not be locally weakly observable, and therefore will neither be locally observable. The system Σ satisfies the observability rank condition if any of the observability matrices are of rank n (recall that $x \in R^n$). The observability matrices are given by:

$$K_j = \begin{bmatrix} L_f^0 dh_j \\ L_f^1 dh_j \\ \vdots \\ L_f^{n-1} dh_j \end{bmatrix} \quad \text{for } 1 \le j \le m \tag{18}$$

or by any combination of n Lie derivatives $L_f^i dh_j$ forming a square matrix of dimension n. The elements of these matrices, $L_f^i dh_j$, are the i^{th} repeated Lie derivative of the j^{th} component of dh with respect to f. Specifically, the Lie derivative of a scalar h with respect to a vector f is a vector field defined by:

$$L_f h = \frac{\partial h}{\partial x} f(x) \tag{19}$$

Similarly the Lie derivative of dh with respect to f is defined by:

$$L_f dh = d(L_f h) = \frac{\partial h}{\partial x}\frac{\partial f}{\partial x} + [\frac{\partial}{\partial x}(\frac{\partial h}{\partial x})^T f]^T \tag{20}$$

The superscript indicates repeated Lie derivatives, which are defined recursively as follows:

$$L_f^0 dh = \frac{\partial h}{\partial x} \tag{21}$$

$$L_f^i dh = L_f^{i-1} dh \cdot \frac{\partial f}{\partial x} + [\frac{\partial}{\partial x}(L_f^{i-1} dh)^T f]^T \tag{22}$$

4 Observability of a Pressure in a Pneumatic Servo System

If, as proposed, the pressure states are not measured, then the output would be the motion of the load, which is given by z and $\dot{z}$. Knowledge of z in time, however, provides all information regarding $\dot{z}$, and therefore in order to simplify the analysis, the measured output is assumed to be displacement only:

$$y = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} z \\ \dot{z} \\ P_a \\ P_b \end{bmatrix} \tag{23}$$

As such, the (scalar) output function $h(x)$ is given as

$$h(x) = [h_1(x)] = z \tag{24}$$

Since the functions f and h must be continuous, each of the eight possible dynamic regimes described by Eqs. (15-16) must satisfy the observability rank condition. Consider first the dynamic regime described by the unchoked charging of chamber a and the choked discharging of chamber b (which, from experimental experience, is a common dynamic regime). Given the model for this dynamic regime, the repeated Lie derivatives (the elements of the observability matrix) will be as follows:

$$L_f^0 dh_1 = \frac{\partial h_1}{\partial x} = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \tag{25}$$

$$L_f^1 dh_1 = \frac{\partial h_1}{\partial x}\frac{\partial f}{\partial x} + [\frac{\partial}{\partial x}(\frac{\partial h_1}{\partial x})^T f]^T = \begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix} \tag{26}$$

$$L_f^2 dh_1 = L_f^1 dh_1 \cdot \frac{\partial f}{\partial x} + [\frac{\partial}{\partial x}(L_f^1 dh_1)^T f]^T = \begin{bmatrix} 0 & 0 & A_a & -A_b \end{bmatrix} \tag{27}$$

$$L_f^3 dh_1 = L_f^2 dh_1 \cdot \frac{\partial f}{\partial x} + [\frac{\partial}{\partial x}(L_f^2 dh_1)^T f]^T = \begin{bmatrix} L_{41} & L_{42} & L_{43} & L_{44} \end{bmatrix} \tag{28}$$

where

$$\begin{aligned} L_{41} = {} & \frac{A_a^2}{(V_{a0} + A_a z)^2}\left(A_a \gamma P_a \dot{z} - A_v \gamma C_d P_s \sqrt{2RT_s} \sqrt{\frac{\gamma}{1-\gamma}\left(\left(\frac{P_a}{P_s}\right)^{\frac{2}{\gamma}} - \left(\frac{P_a}{P_s}\right)^{\frac{1+\gamma}{\gamma}}\right)} \right) \\ & + \frac{A_b^2}{(V_{b0} - A_b z)^2}\left(A_v \gamma C_d P_b \sqrt{RT_b \left(\frac{2\gamma}{1+\gamma}\right)^{\frac{\gamma+1}{\gamma-1}}} + A_b \gamma P_b \dot{z} \right) \end{aligned} \tag{29}$$

$$L_{42} = -\left(\frac{A_a^2 \gamma P_a}{(V_{a0} + A_a z)^2} + \frac{A_b^2 \gamma P_b}{(V_{b0} - A_b z)^2} \right) \tag{30}$$

$$L_{43} = \frac{A_a}{V_{a0} + A_a z}\left(\frac{A_v \gamma \sqrt{T_s} C_d \left(2\left(\frac{P_a}{P_s}\right)^{-1+\frac{2}{\gamma}} - (1+\gamma)\left(\frac{P_a}{P_s}\right)^{-1+\frac{1+\gamma}{\gamma}} \right)}{\sqrt{\frac{2\gamma(\gamma-1)}{R}\left(\left(\frac{P_a}{P_s}\right)^{\frac{2}{\gamma}} - \left(\frac{P_a}{P_s}\right)^{\frac{1+\gamma}{\gamma}} \right)}} - \gamma A_a \dot{z} \right) \tag{31}$$

and

$$L_{44} = \frac{A_b}{V_{b0} - A_b z}\left(A_v \gamma \sqrt{R T_b} C_d \sqrt{\left(\frac{2\gamma}{1+\gamma}\right)^{\frac{\gamma+1}{\gamma-1}}} - \gamma A_b \dot{z} \right) \tag{32}$$

where V_{a0} and V_{b0} are the volumes, respectively, of each cylinder chamber when z=0 (i.e., when the piston is at the midpoint of the cylinder), T_s is the temperature of the supply air, T_b is the temperature of the air in chamber b, and all other variables are as previously defined. The observability matrix generated by $L_f^0 dh_1$, $L_f^1 dh_1$, $L_f^2 dh_1$, and $L_f^3 dh_1$ is thus given by

$$K_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & A_a & -A_b \\ L_{41} & L_{42} & L_{43} & L_{44} \end{bmatrix} \tag{33}$$

Note that this is the only observability matrix for this system in this dynamic regime, and therefore full rank of this matrix is a necessary condition for local weak observability of the pneumatic system. If at any time any of the expressions

$$A_a L_{44} = -A_b L_{43}, \tag{34}$$

$$L_{41} \neq 0 \quad \text{while} \quad L_{42} = L_{43} = L_{44} = 0, \tag{35}$$

or

$$L_{42} \neq 0 \quad \text{while} \quad L_{41} = L_{43} = L_{44} = 0 \tag{36}$$

are satisfied, then the observability matrix will lose rank and the system will not be locally weakly observable (and thus will neither be locally observable). From Eqs. (31) and (32), it can be shown that Eq. (34) is satisfied when $A_v = \dot{z} = 0$, at which point $L_{43} = L_{44} = 0$. Though the expressions given by Eqs. (31-32) describe the specific condition of the unchoked charging of chamber *a* in combination with the choked discharging of chamber *b*, all eight dynamic regimes in which the pneumatic servo actuator can operate will lose local observability when $A_v = \dot{z} = 0$. This condition will occur when the system is at rest, and could additionally occur each time the system reverses direction. Under such circumstances (e.g., during the steady-state portion of tracking a square wave), the nonlinear system is rendered locally unobservable. Further, though the solution to Eq. (34) alone is sufficient to violate local weak observability, the system is additionally rendered unobservable at other points in the operational space, as described by the combinations of Eqs. (29-32) and Eqs. (35-36). These solutions are considerably more complex than the solution of Eq. (34), however, and do not appear to have clear physical interpretations, and as such are not listed here.

Finally, this analysis regarding the observability of pressure contains a few caveats that should be mentioned. First, the fact that Eq. (33) loses rank under certain operating conditions disproves only local observability; thus the system may in fact still retain global observability. Despite this, the authors are not aware of any test for global observability applicable to this type of system, and thus the existence of global observability cannot be assessed. Second, recall that the state equations given by Eqs. (15-16) describe eight possible dynamic regimes. The rank test for local weak observability proposed by Hermann and Krener, however, requires that the state function vector f and the output function vector h be continuous and differentiable with respect to the state (i.e., the repeated Lie derivates must exist), which is not the case for a pneumatic servo system. As such, the (local weak) observability can be analyzed within each dynamic regime, but may be further compromised by the existence of switching between dynamic regimes. Third, the conclusions that have been drawn assumed measurement of output motion (i.e., position) only, while in fact there may be other states or information channels that could render the system observable.

5 Conclusion

This paper investigated the possibility of eliminating pressure sensors in a pneumatic servo actuator through the use of a nonlinear state observer. A nonlinear observability analysis indicated that, given the use of pressure and motion as state variables and motion as output, while regions of local weak observability exist, the system loses local observability at several points in the state space. As such, for most practical purposes, one can conclude that observation of pressure from measurement of motion in a pneumatic servo system is not feasible.

References

[1] Mannetje, J. J., 1981, "Pneumatic Servo Design Method Improves System Bandwidth Twenty-fold," *Control Engineering*, vol. 28, no. 6, pp. 79-83.

[2] Liu, S., and Bobrow, J. E., 1988, "An Analysis of a Pneumatic Servo System and Its Application to a Computer-Controlled Robot," *ASME Journal of Dynamic Systems, Measurement, and Control,* vol. 110, no. 3, pp. 228-235.

[3] Ben-Dov, D. and Salcudean, S.E., 1995, "A Force-Controlled Pneumatic Actuator," *IEEE Transactions on Robotics and Automation*, vol. 11, no. 6, pp. 906-911.

[4] S. R. Pandian, Y. Hayakawa, Y. Kanazawa, Y. Kamoyama and S. Kawamura, 1997, "Practical Design of a Sliding Mode Controller for Pneumatic Actuators", *ASME Journal of Dynamic Systems, Measurement, and Control,* vol. 119, no. 4, pp. 664-674.

[5] Bobrow, J., and McDonell, B., 1998, "Modeling, Identification, and Control of a Pneumatically Actuated, Force Controllable Robot," *IEEE Transactions on Robotics and Automation,* vol. 14, no. 5, pp. 732-742.

[6] Richer, E. and Hurmuzlu, Y., 2000, "A High Performance Pneumatic Force Actuator System: Part II-Nonlinear Control Design" *ASME Journal of Dynamic Systems, Measurement, and Control*, vol. 122, no. 3, pp. 426-434.

[7] Acarman, T., Hatipoglu, C., Ozguner, U., 2001, "A Robust Nonlinear Controller Design for a Pneumatic Actuator", *Proceedings of the American Control Conference*, vol. 6, pp. 4490-4495.

[8] Shearer, J. L., 1956, “Study of Pneumatic Processes in the Continuous Control of Motion with Compresses Air – I,” *Transactions of the ASME*, vol. 78, pp. 233-242.

[9] Shearer, J. L., 1956, “Study of Pneumatic Processes in the Continuous Control of Motion with Compresses Air – II,” *Transactions of the ASME*, vol. 78, pp. 243-249.

[10] Shearer, J. L., 1957, “Nonlinear Analog Study of a High-Pressure Servomechanism,” *Transactions of the ASME*, vol. 79, pp. 465-472.

[11] Burrows, C. R., 1972, *Fluid Power Servomechanisms*, Butler & Tanner Ltd, London.

[12] McCloy, D., and Martin, H., 1980. *Control of Fluid Power*, Ellis Horwood Limited, Chichester, England.

[13] Richer, E. and Hurmuzlu, Y., 2000, “A High Performance Pneumatic Force Actuator System: Part I-Nonlinear Mathematical Model” *ASME Journal of Dynamic Systems, Measurement, and Control*, vol. 122, no. 3, pp. 416-425.

[14] Hermann, R. and Krener, A.J., 1977, “Nonlinear Controllability and Observability”, IEEE Transactions on Automatic Control, vol. 22, no 5, pp. 728-740.