Nonlinear Model-Based Control of Pulse Width Modulated Pneumatic Servo Systems

This paper presents a control methodology that enables nonlinear model-based control of pulse width modulated (PWM) pneumatic servo actuators. An averaging approach is developed to describe the equivalent continuous-time dynamics of a PWM controlled nonlinear system, which renders the system, originally discontinuous and possibly non-affine in the input, into an equivalent system that is both continuous and affine in control input (i.e., transforms the system to nonlinear control canonical form). This approach is applied to a pneumatic actuator controlled by a pair of three-way solenoid actuated valves. The pneumatic actuation system is transformed into its averaged equivalent control canonical form, and a sliding mode controller is developed based on the resulting model. The controller is implemented on an experimental system, and the effectiveness of the proposed approach validated by experimental trajectory tracking.

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1 Introduction

The servo control of pneumatic actuators is typically implemented by utilizing some type of servo valve to control the airflow into and out of the respective sides of a pneumatic cylinder. Several researchers have studied the design and control of such systems, including Shearer [1–3], Mannette [4], Wang et al. [5], Bobrow and McDonell [6], and Richer and Hurmzu [7,8], whose studies involved servo control via spool-type four-way servo valves, and Jacobsen et al. [9], Ben-Dov and Salcudean [10], and Henri et al. [11], whose studies involved control via flapper or jet-pipe-type servo valves. In such systems, the cost of the servo valve in nearly all cases dominates the cost of the actuator.

Pulse width modulated (PWM) control offers the ability to provide servo control of pneumatic actuators at a significantly lower cost by utilizing binary solenoid valves in place of proportional servo valves. In a pneumatic servo system controlled by proportional servo valves, the power delivered to the pneumatic actuator is metered by continuously varying the flow resistance of the valve, which in turn continuously varies the fluid mass flow rate into and out of the respective sides of the cylinder. In a PWM-controlled system, the power delivered to the actuator is metered discretely by delivering packets or quanta of fluid mass via a valve that is either completely on or completely off. If delivery of these packets of mass occurs on a time scale that is significantly faster than the system dynamics (i.e., dynamics of the actuator and load), then the system will respond in essence to the average mass flow rate into and out of the cylinder, in a manner similar to the continuous case. As is the case with control via servo valves, several researchers have investigated the use of PWM control of solenoid on/off valves for the servo control of pneumatic actuators. In particular, Noritsugu [12,13], Kunt and Singh [14], Ye et al. [15], and Shih and Hwang [16] incorporate principally heuristic approaches for the PWM control of pneumatic servo systems. Though such approaches afford a level of control, they do not provide the stability or performance guarantees of approaches developed within a more rigorous analytical framework. The work of van Varseveld and Bone [17] proposes the use of discrete time linear controllers for the PWM control of pneumatic servo systems. Barth et al. [18] utilize a linear state-space averaging technique to enable the design of a linear compensator via a loop-shaping approach that provides a prescribed degree of stability robustness in addition to a desired closed-loop bandwidth. Both of these prior works treat the PWM control of pneumatic servo systems within the context of linear control. Because of their highly nonlinear nature, pneumatic servo systems are better addressed by the use of nonlinear model-based control techniques. Paul et al. [19] proposed a switching controller (not technically a PWM controller) based on a “reduced-order” nonlinear model that provides stability in the sense of Lyapunov. The reduced-order aspect of their approach, however, requires simplifying assumptions that cannot accommodate the full nonlinear character of a pneumatic servo system. In particular, they neglect the nonlinearity in the chamber pressure dynamics, the change in pressure boundary conditions that results when switching the direction of control effort (i.e., the upstream and downstream pressures switch from supply and chamber, respectively, to chamber and atmosphere), and the distinction between the choked and unchoked flow regimes through the solenoid valves. These combined effects constitute significant nonlinear behavior in such systems.

Unlike these prior works, this paper presents a method for nonlinear model-based PWM control of a pneumatic servo actuator based on the full nonlinear model of such systems. Specifically, this paper extends the authors’ previously published averaging techniques, which were developed in the context of linear systems [18], to nonlinear systems. The nonlinear averaging technique is then utilized as the basis for the development of a PWM-based sliding mode approach to the control of pneumatic servo systems, which is based on the full (i.e., non-reduced-order) nonlinear description of such systems. The controller is implemented on a single-degree-of-freedom pneumatic servo system, and the effectiveness of the method verified by experimental trajectory tracking.

2 Average Model-Based PWM-Control of Nonlinear Systems

A pulse width modulated control system meters the power delivered to an actuator from a power source in discrete packets, as opposed to the continuous delivery of power characteristic in a
continuous control system [i.e., those treated in [1−11]]. The “packets” of power delivered by a PWM system, however, are, in essence, averaged by the dynamics of the system being controlled. As such, the resulting dynamics on the characteristic time scale of interest for the closed-loop system can be described by the average dynamics of such systems. Consider a general nonlinear dynamic system given in regular form as

\[ x^{(n)} = f(x, u) \]  

(1)

where \( x^{(n)} \) is the \( n \)th derivative of \( x, x \in \mathbb{R}^n \) is a vector of the continuous states of the system (i.e., all lower derivatives of \( x \)), \( u \in \mathbb{R}^m \) is the vector of control inputs, and \( f(\cdot, \cdot) \) describes the form of the system dynamics. In a PWM-controlled system, the elements of the control vector \( u_i \) are binary variables, such that they can only assume values of 0 or 1. Consider the case of \( p \) allowable combinations of the control vector \( u_i \) each referred to as input mode \( i, \) such that \( u_i : i \in \{1, 2, \ldots, p\} \). Since the elements of \( u_i \) are binary, selection of an input mode effectively selects a subset of the dynamics \( f(x, u_i) \). If \( f(x) \) describes the dynamics corresponding to the input \( u_i \), then the system dynamics for a PWM-controlled system can assume \( p \) distinct forms, which correspond to its \( p \) distinct input modes. The dynamics of the system operating in each respective mode can be written as

\[ x^{(n)} = f_i(x) \quad \text{for} \quad 1 \leq i \leq p \]  

(2)

Within each PWM period, the control input can switch between modes 1 through \( p \), where the fraction of the PWM period that any given mode is active is the duty cycle of that mode, or modal duty cycle, denoted as \( d_i \). These modal duty cycles can be written in vector form as

\[ D = [d_1, d_2, \ldots, d_p]^T \]  

(3)

where the 1-norm of this vector must satisfy

\[ \|D\| = 1 \]  

(4)

which simply indicates that the duration of modes active must comprise the full duration of the PWM period. Collecting the system dynamics for each mode in the vector

\[ F = [f_1, f_2, \ldots, f_p]^T \]  

the average system dynamics can be described by

\[ x^{(n)} = F^TD \]  

(6)

In the case of \( p=2 \) (i.e., the minimum possible number of input modes), the input/output relationship is uniquely specified (i.e., it is the PWM equivalent of a single-input, single-output system). In this case, because of the constraint (4), the modal input vector can be written in terms of a single duty cycle as

\[ D = \begin{bmatrix} d \\ 1-d \end{bmatrix} \]  

(7)

and the state averaged model of this system written as

\[ x^{(n)} = f_1d + f_2(1-d) \]  

(8)

The form of (8) can be recast in control canonical form, affine in the new continuous control variable \( d \) as

\[ x^{(n)} = \tilde{f}(x) + \tilde{b}(x)d \]  

(9)

where \( \tilde{f}(x) = f_2(x), \tilde{b}(x) = f_1(x) - f_2(x) \), and the input is confined to a saturated range \( d \in [0,1] \). Since most control systems are generally symmetric in the control effort, in many cases it may be preferable to transform the control input \( d \) into a symmetric input \( u \) centered about zero (i.e., \( u \in [-1,1] \), though the amplitude may be other than unity if so desired). A simple linear transformation for the case \( u \in [-1,1] \) can be expressed as

\[ u = 2d - 1 \]  

(10)

such that (9) can be written as

\[ x^{(n)} = \tilde{f}(x) + \tilde{b}(x)u \]  

(11)

where the continuous input is confined to the symmetric range \( u \in [-1,1] \). Note that a nonlinear transformation could be used in place of (10) such that \( u \in (-\infty, \infty) \), if so desired. Equation (11) expresses a system dynamics, originally influenced by discontinuous control inputs, into an equivalent system that is affine in its single, continuous control input (i.e., the nonlinear system has been transformed into a control canonical form). Such a form is well suited to many nonlinear control approaches (e.g., sliding mode control, integrator back-stepping control).

In the case of \( p>2 \) (i.e., three or more input modes), the mapping from the control inputs to system output \( x \) is nonunique and, as such, constraints must be added in order to provide a unique dynamics (i.e., the case of \( p>2 \) is the PWM equivalent of a multi-input, single-output system). Specifically, \( p-2 \) constraints must be added to constraint (4) so that the vector of modal inputs can be described as a function of a single input (as described for the two-mode case by Eq. (7)). These constraints can be written in a general form as

\[ D = \gamma(d) \]  

(12)

where \( \gamma(\cdot) \in \mathbb{R}^p \). Given the constraints expressed by (12), along with Eqs. (6) and (10), the system dynamics can be written in control canonical form as a function of the symmetric, continuous control input \( u \in [-1,1] \) as

\[ x^{(n)} = \tilde{f}(x) + \tilde{b}(x)u \]  

(13)

where the exact forms of \( \tilde{f} \) and \( \tilde{b} \) depend on the form of Eq. (12). Thus, a nonlinear system, originally discontinuous and possibly nonaffine in the input, can be transformed into an equivalent system that is both continuous and affine in the single continuous control input. Various nonlinear model-based techniques may be applied directly to the resulting equivalent nonlinear control canonical model. Based on this approach, the following two sections develop a nonlinear model and sliding mode controller for a pneumatic servo system.

3 Nonlinear Averaged Model of a PWM-Controlled Pneumatic Servo System

In the case of a pneumatic servo system controlled by solenoid valves, the (previously mentioned) discrete inputs to the nonlinear system are the valve spool (or poppet) positions, and the continuous output is the motion of the load. In order to convert the discontinuous system inputs to an equivalent continuous input, the behavior of the system in each discrete mode must first be described. This section of the paper derives a model of the pneumatic servo system, defines and describes the discrete modes of operation of this system, and finally derives the equivalent description from (continuous) input to motion based on the averaging methods previously described.

Assuming air is a perfect gas undergoing an isothermal process, the rate of change of the pressure inside each chamber of the cylinder can be expressed as

\[ \dot{P}_{(a,b)} = \frac{RT}{V_{(a,b)}} (\tilde{m}_{\text{in}(a,b)} - \tilde{m}_{\text{out}(a,b)}) = \frac{P_{(a,b)}}{V_{(a,b)}} \dot{V}_{(a,b)} \]  

(14)

where \( P_{(a,b)} \) is the absolute pressure inside each side of the cylinder, \( \tilde{m}_{\text{in}(a,b)} \) and \( \tilde{m}_{\text{out}(a,b)} \) are the mass flow rates into and out of each side of the cylinder, \( R \) is the universal gas constant, \( T \) is the fluid temperature, and \( V_{(a,b)} \) is the volume of each cylinder chamber. Based on isentropic flow assumptions, the mass flow rate though a valve orifice with effective area \( A_e \) for a compressible substance will reside in either a sonic (choked) or subsonic (unchoked) flow regime.
where \( C_f \) is the discharge coefficient of the valve, \( P_u \) and \( P_d \) are the upstream and downstream pressures, respectively, \( k \) is the ratio of specific heats, \( C_e \) is the pressure ratio that divides the flow regimes into unchoked and choked flow, and \( A_v \) is the cross-sectional area of the valve orifice. Assuming a combined inertial and viscous load, the motion dynamics of the system shown in Fig. 1 can be written as

\[
M \ddot{x} + B \dot{x} = P_u A_u - P_d A_b - P_{\text{atm}} A_f,
\]

(16)

where \( M \) is the payload plus the piston and rod assembly mass, \( B \) is the viscous friction coefficient, \( A_u \) and \( A_b \) are the effective areas of each side of the piston, and \( A_f \) is the cross-sectional area of the piston rod. The total dynamics from the valve area as input (which is an algebraic function of the spool position) to the motion output is given by the combination of Eqs. (14)–(16)

\[
\begin{align*}
\dot{m}(P_u, P_d) &= \begin{cases} \sqrt{k \frac{2}{RT} \frac{2}{k+1} \frac{1}{(k+1)^{k+1}}} C_f P_u A_u & \text{if } P_d \leq C_e \text{ (choked)} \\
\sqrt{-\frac{2k}{RT(k-1)} \frac{1}{1-(P_d/P_u)^{k-1}} (P_d/P_u)^{1/k}} C_f P_u A_u & \text{otherwise (unchoked)}
\end{cases}
\end{align*}
\]

(15)

Mode 2: \( u_2 = A_u [0 \ 1 \ 0 \ 1]^T \)

Mode 3: \( u_3 = A_u [0 \ 1 \ 1 \ 0]^T \)

Mode 4: \( u_4 = A_u [1 \ 1 \ 1 \ 0]^T \)

(20)-(22)

where mode 1 corresponds to charging side \( a \) and discharging \( b \), mode 2 corresponds to discharging both, mode 3 corresponds to charging side \( b \) and discharging \( a \), mode 4 corresponds to charging both, and \( A_u \) is the valve area of the open valves. Since, in the presence of mode 2, mode 4 does not offer significant utility with respect to tracking control, mode 4 is not considered further (i.e., \( d_4 = 0 \)). Thus, the controller can assume any of the first three modes, and as such, the modal duty cycle vector (as defined by (3)), is given by

\[
D = [d_1 \ d_2 \ d_3]^T
\]

(23)

Since the number of input modes for this system is three, a single constraint equation is required to properly constrain the system (as described in Sec. 2). In this case, the constraint can be formulated by observing that, within a single PWM period, the controller will assume either a combination of modes 1 and 2, which corresponds to a control effort in one direction, or a combination of modes 2 and 3, which corresponds to a control effort in the opposite direction. Combining this constraint with the constraint of Eq. (4), the modal input vector can be written as a function of a single input (i.e., as in Eq. (7)) as

\[
D^T = \begin{cases} 
[2d - 1 \ 2(1 - d) \ 0] & \text{if } d \geq 0.5 \\
[0 \ 2d - 1 \ -2d] & \text{otherwise}
\end{cases}
\]

(24)

where \( d \in [0,1] \). Utilizing the transformation of Eq. (10) to provide control symmetry, the duty cycle vector can be rewritten as

\[
D^T = \begin{cases} 
[1 \ a \ -a] & \text{if } a \geq 0 \\
[0 \ 1 + a \ -a] & \text{otherwise}
\end{cases}
\]

(25)

where \( a \in [-1,1] \). Based on Eqs. (15)–(21) and (25), the corresponding average model can be derived following the method presented in Sec. 2 as shown in the Appendix, which yields the following description:

\[
\bar{x} = \begin{cases} 
f(x) + b^+(x)u & \text{if } u \geq 0 \\
f(x) + b^-(x)u & \text{if } u < 0
\end{cases}
\]

(26)

where

\[
f(x) = \frac{A_u}{M} \left[ \frac{RT}{V_a} m(P_u, P_{\text{atm}}) + \frac{P_u}{V_a} \dot{V}_a \right] + \frac{A_b}{M} \left[ \frac{RT}{V_b} m(P_d, P_{\text{atm}}) \right]
\]

\[
\dot{b}^+(x) = \frac{A_u}{M V_a} \left[ m(P_u, P_u) + m(P_d, P_{\text{atm}}) \right]
\]

(27)-(29)

where the state vector consists of the pressure in each side of the cylinder, along with the position, velocity, and acceleration of the load.

Fig. 1 Solenoid valve controlled pneumatic servoactuator driving an inertial load.
Note that, though not explicitly shown, the volume and rate of change of volume of each chamber are straightforward functions of the state (i.e., load displacement and velocity, respectively), while the mass flow rates are functions of the measured pressures, as given by Eq. (15). Equations (26)–(29) describe the equivalent dynamics of the solenoid-actuated pneumatic servo system in non-linear control canonical form. As in a standard pneumatic servo system (i.e., one controlled via proportional servo valves), the dynamics of the system described in (26)–(29) depend on the sign of the control input (i.e., the pressure boundary conditions that drive the inlet or outlet mass flow rates depend on whether the respective chambers are charging or discharging, as determined by the sign of the input). That is, the split form of Eq. (26) is a by-product of the pneumatic system dynamics, and not a result of the proposed PWM approach.

4 Pulse-Width-Modulated Sliding Mode Control

Having expressed the PWM system dynamics in a continuous input control canonical form, a sliding mode control approach can be applied to the control of the system. Selecting an integral sliding surface as

\[ s = \left( \frac{d}{dt} + \lambda \right) \int_0^t e(\tau) d\tau \]  

(31)

where \( e = x - x_{\text{desired}} \) and \( \lambda \) is a control gain, a robust control law can be developed based on a standard sliding mode control approach, where the equivalent control component is derived by solving for the input for the case \( \dot{x} = 0 \), which gives

\[ u_{eq} = \begin{cases} x_d - f(x) - 3\lambda \dot{x} - 3\lambda^2 \dot{\dot{x}} - \lambda^3 e \\ b_{\text{eq}}(x) \end{cases} \]  

(32)

where the superscript + or − denotes the corresponding value when \( u \) is positive or negative (i.e., as previously mentioned, the dynamics are sensitive to the sign of the input). The robust control law is obtained by adding a robustness component, such that

\[ u = u_{eq} - K \text{sgn}(s) \]  

(33)

where the robustness gain \( K \) is time variant according to

\[ K \text{sliding} = K^{\text{sliding}}(F + \eta) + (\beta + 1)(F_d - f(x) - 3\lambda \dot{x} - 3\lambda^2 \dot{\dot{x}} - \lambda^3 e) \]  

(34)

where \( \beta \approx (|b_{\text{eq}}^+|/|b_{\text{eq}}^-|)^{1/2} \), and \( F = \alpha |f| \), where \( \alpha \) characterizes the magnitude of the uncertainty in the homogeneous component of the system model (i.e., \( \alpha = 0.1 \) implies a 10% uncertainty in the magnitude of \( f(x) \)). A greater degree of model uncertainty effectively requires a higher gain. As previously mentioned, the dependence of the dynamics on the sign of the input is a characteristic of pneumatic servo systems. This dependence is accommodated by using the equivalent control component that corresponds to the proper direction of control effort, so that

\[ u = u_{eq} - K \text{sgn}(s) \]  

(35)

where

\[ u_{eq} = \begin{cases} u_{eq}^+ \text{ if } u \geq 0 \\ u_{eq}^- \text{ otherwise} \end{cases} \]  

(36)

and

\[ K = \begin{cases} K^+ \text{ if } u \geq 0 \\ K^- \text{ otherwise} \end{cases} \]  

(37)

That is, if the resultant control effort is positive, the controller will be charging chamber \( a \) and discharging chamber \( b \), and as such, the equivalent control law and the robustness gain corresponding to this assumption must be used. If the resultant control effort is negative, the equivalent control law and robustness gain corresponding to this assumption must be used. If the resultant control effort is negative, the equivalent control law and robustness gain corresponding to this assumption must be used.

5 Experimental Results

Experiments were conducted to validate the proposed control approach. The experimental setup, which is shown schematically in Fig. 1, incorporated a 2.7 cm (1–1/16 in.) I.D., 10 cm (4 in.) stroke pneumatic cylinder (Numatics 1062D04-04A) that was used to position a 10 kg mass. The airflow was controlled by a pair of pilot-assisted three-way solenoid-actuated valves (SMC VQ1200H-5B) operating at a PWM frequency of 25 Hz. The system was supplied with air at an absolute pressure of 584 kPa (85 psig). Implementation of the control law requires measurement of the model states, which is required specifically for the computation of \( f(x) \) and \( b_{\text{eq}}(x) \) in Eqs. (32) and (34), and for computation of the error used in Eqs. (31)–(34). For these purposes, pressure transducers (Omega PX202-200GV) were used to measure the pressure in each cylinder chamber (i.e., \( P_a \) and \( P_b \)), and a linear potentiometer (Midori LP-100F) was used to measure the linear position of the load \( x \). The velocity and acceleration of the load \( \dot{x} \), both of which are also states, were provided via filtered differentiation of the measured position with a filter roll-off at 25 Hz. The model and controller parameters used in the experiments are listed in Table 1. The tracking performance was assessed by sinusoidal tracking at various frequencies with a peak-to-peak motion amplitude of 40 mm (i.e., 40% of the full-scale cylinder motion). Specifically, Figs. 2–4 show the measured motion of the 10 kg mass for a commanded sinusoidal motion of 0.25, 0.5, and 1.0 Hz, respectively, along with the tracking error for each case. As shown, the control approach provides effective tracking of continuous motion via solenoid on/off valves. Tracking performance was degraded at higher frequencies, presumably due to some combination of choked flow through the valves (which limits the actuation power) and their limited switching response time. Regarding the latter, as with any PWM-controlled system, the closed-loop system bandwidth is limited to approximately an order of magnitude below the PWM switching frequency, which in this case was 25 Hz, limited by the bandwidth of the valves. Thus, even without the mass flow saturation (i.e., choked flow), it is unlikely that this system could track frequen-
cies much greater than 2 Hz. Figure 5 shows the (continuous, symmetric) control command, as generated by Eqs. (31)–(37), corresponding to the 1.0 Hz sinusoidal tracking shown in Fig. 4. Note that the control command is subject to a 25 Hz zero-order hold, corresponding to the 25 Hz PWM period. Finally, Fig. 6 shows the same control command for two cycles of the sinusoidal tracking, and Figs. 7 and 8 show the resulting discrete valve commands corresponding to the control command of Fig. 6. Recall that valve a is opened for a duty cycle corresponding to the control command when the command is positive, and valve b is opened for a duty cycle corresponding to the control command when the command is negative, as described by Eq. (25).

6 Conclusion

This paper presented a control approach capable of providing nonlinear servo control via relatively inexpensive on/off solenoid valves. A nonlinear model averaging approach was developed that enabled the use of a full (non-reduced-order) nonlinear model-based control. This averaging method was applied to a PWM-controlled pneumatic servo system, followed by the development of a sliding mode controller for that system. The proposed controller was implemented on an experimental setup and shown via the resulting tracking performance to provide effective control of a continuous motion command.

Appendix

The single-input, single-output average model given by Eqs. (26)–(29) is derived by first defining the modal system dynamics. Based on Eqs. (18) and (19), the mass flow rates corresponding to mode 1 are given by

\[ m_{\text{in},a} = \dot{m}(P_s, P_a) \]
\[ m_{\text{out},a} = 0 \]
\[ m_{\text{in},b} = 0 \]
\[ m_{\text{out},b} = \dot{m}(P_p, P_{\text{atm}}) \]  

where \( \dot{m}(-) \) is given by Eq. (15). Substituting these into the generalized system dynamics of Eq. (17) yields the system dynamics of mode 1, given by...
Based on Eqs. (18) and (20), the mass flow rates corresponding to mode 2 are given by

\[ m_{\text{in},a} = 0 \]
\[ m_{\text{out},a} = m(P_a \cdot P_{\text{atm}}) \]
\[ m_{\text{in},b} = 0 \]
\[ m_{\text{out},b} = m(P_b \cdot P_{\text{atm}}) \]  \hspace{1cm} (A3)

Substituting these into the generalized system dynamics of Eq. (17) yields the system dynamics of mode 2, given by
and the modal system dynamics vector as
\[ \dot{x} = f(x) \]
and substituting in Eq. (A9) yields
\[ f(x) = f_1 + f_2 + f_3 \]
Rearranging this in the standard control canonical form yields
\[ x = \begin{cases} f_1 u + f_2 (1 - u) & \text{if } u \geq 0 \\ f_1 (1 - u) - f_2 u & \text{otherwise} \end{cases} \]
Thus, the respective terms in the average system dynamics of Eq. (26) are given by
\[ f(x) = f_2 \]
\[ b(x) = f_1 - f_2 \]
\[ b^*(x) = f_2 - f_3 \]
References