A piezoelectric-driven inchworm locomotion device

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Abstract

An inchworm locomotion device is presented that can walk on smooth terrain. It consists of a piezoelectric unimorph and two custom-designed legs. A mathematical model is formulated by superimposing the compliant and rigid-body components of the motion. The model is subsequently utilized to evaluate the ground reaction forces and horizontal velocity. Numerical simulation is performed to select the geometry parameters that improve the motion efficiency. An inchworm prototype was tested to verify the theoretical predictions. © 2001 Elsevier Science Ltd. All rights reserved.

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1. Introduction

The authors are currently developing small locomotion systems that are capable of efficient walking. A critical problem in designing autonomous small-scale walkers is power consumption since the independent control of several legs is energetically expensive while the energy supplied by typical electrochemical batteries is small. An alternative approach, called elastodynamic locomotion, is employed as a means of increasing the otherwise-limited range of the walking systems. It consists of designing lightly-damped elastic skeletal structures that exhibit high motion efficiency when excited at an appropriate resonance. Unlike conventional-scale machine control, the actuator only excites the open-loop dynamics of the skeleton while there is no control needed for the motion of the robot limbs. A similar approach was followed by Babitsky and Chitayev [1] in designing a high-speed resonant robot.
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**Subscripts**

- $a$: actuator
- $b$: beam
- $i, k$: counters
- $h$: horizontal
- $v$: vertical
- $0$: initial value
- $1, 2$: leg identifier

**Superscripts**

- $1, 2$: cycle phase
- $c$: compliant
- $e$: equivalent
- $rb$: rigid-body
The paper presents an inchworm locomotion device that is extremely simple in terms of mechanical structure, actuation and motion.

The device consists of a unimorph piezoelectric actuator that acts both as a structural component and excitation source and two custom-designed legs. The legs can either roll, which will transport the device ahead, during an active phase, or drag on the ground and regain position for a subsequent active stroke, during a passive phase. The two legs perform opposite tasks at one time but, overall, they produce a low-speed unidirectional motion. The design is functionally similar to the walking tractor described by Thring [2] and the legged-wheel robot investigated by Maza et al. [3].

A mathematical model is formulated that addresses several topics. The bending moment that is produced at the actuator’s ends is first related to the piezoelectric-induced strain. The overall displacement of the inchworm’s center of gravity is then evaluated by superimposing the elastic (bending) deformation to the rigid-body motion. This quasi-static model allows analyzing the geometry of motion without including the forcing effects. The coupling between elastic deformation and rigid-body motion was analyzed, amongst others, by Hac [4] who included the links longitudinal deformation into the model, Yu [5] who studied a flexible robot by using a continuous-displacement field and Carrera and Serna [6] who utilized the inverse finite element dynamics to describe the motion of plane flexible robots.

An inverse dynamic model is then formulated in order to evaluate the ground reaction forces and the horizontal velocity of the inchworm center of gravity. The ground reaction forces are usually determined experimentally by using force plates – Mizoguchi and Calame [7], ink-pads technology – Rafferty and Bell [8] or video/electromagnetic tracking – Kobayashi et al. [9].

Numerical simulations are performed based on the mathematical model and conclusions are drawn regarding the selection of geometry configuration that will maximize the motion capability of a robot.

An inchworm prototype was designed that exhibited a plane unidirectional ‘falling-ahead’ motion. The experimental results confirmed the model predictions.

2. Simplified analysis of motion

A simple design is employed, in terms of actuation, mechanical structure and motion pattern, to illustrate the elastodynamic locomotion concept.

The inchworm locomotion device that is schematically illustrated in Fig. 1 consists of a body – the piezoelectric unimorph, two custom-designed legs and their corresponding clamps that attach the legs to the body and allow adjustment of leg inclination and length. The actuator produces bending vibrations that are transmitted to the legs, which interact with the ground and set the inchworm into a unidirectional motion. The inclination in both body and legs is fundamental in producing the motion.

One motion cycle is composed of two phases. During the first phase the front leg performs a clockwise rolling on the ground while the rear leg drags ahead when the unimorph bends by increasing its curvature up to a maximum. The legs interchange their roles during the second phase when the unimorph bends back and recovers its original shape. The overall result is a unidirectional motion of the inchworm.
3. Mathematical model

A mathematical model is formulated comprising the following sections:
- Forcing model: Relates the electrical input (voltage) to the mechanical output (bending moment).
- Geometric model: Predicts the inchworm’s center of gravity motion without including inertia and time.
- Dynamic model: Evaluates the ground reaction forces and horizontal velocity of the inchworm’s center of gravity by including inertia and time.

3.1. Forcing model

The main component of the inchworm is a THIn-layer composite UNimorph ferroelectric DrivER and sensor (THUNDER), developed at NASA Langley Research Center and manufactured by Face International Corporation [12]. It consists of a piezoceramic patch that is laminated and prestressed between layers of materials (baking) such as: aluminum, stainless steel or beryllium, bonded with a soluble imide adhesive, as indicated by Wise [13] and Mossi et al. [14]. Internal stresses are subsequently induced in the structure through thermal treatment.

A mathematical model is formulated to express the bending moment that is produced at the unimorph ends by the electrical input under simplified boundary conditions.

The following assumptions are applied:
- The actuator patch is considered to only induce strain in the longitudinal direction.
- The end effects caused by the actuator (at locations where the strain field vanishes) are ignored.
- The metal foils and actuator patch are modeled as Euler–Bernoulli beams.
- The strain is considered to vary linearly along the composite beam’s cross-section.
- The bonding is considered ideal (zero bonding layer thickness).
- The whole unimorph is assimilated to a small-curvature beam which is subjected to bending.
The derivation proposed here is similar to that followed by Rees [15] in analyzing thermally-induced strains.

When actuation is applied, the patch will tend to increase/decrease its length and this will produce bending of the composite beam, as illustrated in Fig. 2. The equations that express force/moment equilibrium and strain compatibility at the two interfaces are:

\[
F_{b1} = F_a + F_{b2},
\]

\[
M_{b1} + M_a + M_{b2} = F_{b1} \left( \frac{h_a + h_{b1}}{2} + \frac{h_a + h_{b2}}{2} \frac{F_{b2}}{F_{b1}} \right),
\]

\[
\frac{F_{b1}}{E_b A_{b1}} + \frac{M_{b1} h_{b1}}{2E_b I_{b1}} = -\frac{F_a}{E_a A_a} - \frac{M_a h_a}{2E_a I_a} + \varepsilon_{a,0},
\]

\[
-\frac{F_a}{E_a A_a} + \frac{M_a h_a}{2E_a I_a} + \varepsilon_{a,0} = -\frac{F_{b2}}{E_b A_{b2}} - \frac{M_{b2} h_{b2}}{2E_b I_{b2}},
\]

where

\[
\varepsilon_{a,0} = d_{31} \frac{V}{h_a}
\]

and \(d_{31}\) is the dielectric constant.

By assuming that the curvature radii of the patch and beam layers are approximately equal yields

\[
\frac{E_a I_a}{M_a} = \frac{E_b I_{b1}}{M_{b1}} = \frac{E_b I_{b2}}{M_{b2}}.
\]

Eqs. (1) and (3) form a system of six-algebraic equations, which can be solved for the unknowns \(F_a, F_{b1}, F_{b2}, M_a, M_{b1}\) and \(M_{b2}\).

The maximum bending moment that is generated in the beam at the patch extremities is

\[
M_f = M_{b1} + M_a + M_{b2} = C_f \varepsilon_{a,0}
\]

where \(C_f\) is expressed in Eqs. (A.1).

Fig. 2. Composite cross-section of the unimorph.
3.2. Geometric analysis of motion

The formulation developed in this sub-section assumes ideal geometric deformation and displacement. The purpose is to provide a tool that enables primary selection of the inchworm geometry. The legs are assumed compliant and the unimorph is considered to deform by always preserving its circular shape and length. The leg configuration is pictured in Fig. 3(a). The net motion of the inchworm can be described by superimposing the elastic deformation of the bending unimorph and limbs to the rigid-body rotation of the whole structure around the ground/leg contact point.

3.2.1. Geometry of deformation

A simple model is developed by only using geometric parameters. The following assumptions are employed throughout this sub-section:

- The robot exhibits a low-speed motion and therefore inertia is neglected at this stage.
- One leg performs one single pure motion at a time which is either rolling or sliding.
- The legs are modeled as small-deformation Euler–Bernoulli elastic beams.

In order to simplify the calculus, the original legs are substituted with straight legs along lines connecting the ground contact point and the points of attachment to the unimorph ends, as illustrated in Fig. 3(b). The lengths and angles of the equivalent legs are:

\[ l_{1k}^2 = (h_1 + H_1)^2 + b_1^2 - 2r_1[(h_1 + H_1) \cos \alpha_{1k} - (1 + \sin \alpha_{1k})(b_1 - r_1)] , \]

\[ \begin{align*}
\alpha_{1k} &= A \tan \left[ \frac{(h_1 + H_1) \sin \alpha_{1k} + (b_1 - r_1) \cos \alpha_{1k}}{(h_1 + H_1) \cos \alpha_{1k} - (b_1 - r_1) \sin \alpha_{1k} + r_1} \right] , \\
\end{align*} \]

\[ l_{2k}^2 = (h_2 + H_2)^2 + b_2^2 - 2r_2[(h_2 + H_2) \cos \alpha_{2k} - (1 - \sin \alpha_{2k})(b_2 - r_2)] , \]

\[ \begin{align*}
\alpha_{2k} &= A \tan \left[ \frac{(h_2 + H_2) \sin \alpha_{2k} - (b_2 - r_2) \cos \alpha_{2k}}{(h_2 + H_2) \cos \alpha_{2k} + (b_2 - r_2) \sin \alpha_{2k} + r_2} \right] , \quad k = 0 \rightarrow n \]

for the front and rear leg, respectively. The subscript \( k \) indicates a generic time instant.

![Fig. 3. (a) Leg configuration; (b) geometry of equivalent legs.](image-url)
According to Fig. 3(b) the angle $\beta_k$ can be expressed as

$$\beta_k = \arcsin \left( \frac{l_{2k} \cos \varphi_{2k} - l_{1k} \cos \varphi_{1k}}{L_{ck}} \right),$$

where

$$L_{ck} = 2 \frac{L}{\partial_k} \sin \frac{\theta_k}{2}.$$  

(6)  

Fig. 3(b) also enables to express the constant angles $\gamma$ and $\delta$:

$$\gamma = \frac{\pi}{2} - \varphi_{1k} + \beta_k + \frac{\theta_k}{2},$$

$$\delta = \frac{\pi}{2} + \varphi_{2k} - \beta_k + \frac{\theta_k}{2}.$$  

(8)

Substitution of Eqs. (5)–(7) into (8) produces a system of two trigonometric equations in $\varphi_{1k}$ and $\varphi_{2k}$ that cannot be solved analytically. Back substitution of $\varphi_{1k}$ and $\varphi_{2k}$ into Eqs. (5) allows calculation of the length and inclination angle for the equivalent legs. Eqs. (5)–(8) define a specific state of the robot by means of one single variable ($\theta$ for instance) and this demonstrates that the robot is a single-degree-of-freedom mechanism.

The stiffness of the equivalent legs can be determined by equating the strain energies of the original and equivalent legs.

The strain energy for a structural member in bending is

$$U_b = \frac{1}{2} \int \frac{M_b^2}{EI} \, dl.$$  

(9)

For a fixed-free configuration the equivalent moments of inertia for the front and second leg are, respectively:

$$I_{1e} = \frac{l_{1,0}}{r_1 \left( \frac{\pi}{2} + \varphi_{1,0} \right) + h_1 + H_1 + l_1} I_1,$$

$$I_{2e} = \frac{l_{2,0}}{r_1 \left( \frac{\pi}{2} - \varphi_{2,0} \right) + h_2 + H_2 + l_2} I_2.$$  

(10)

In Eqs. (10) $I_1$ and $I_2$ are the moments of inertia of the original constant-cross-section legs.

The forward motion is produced thus solely by rolling from the front leg, during the first half of a cycle and from the rear leg, during the second one. Fig. 4 illustrates the three main positions of the inchworm over one cycle, namely: the start of a cycle, end of first half (start of the second one) and end of the second half (start of the first one). Two different configurations correspond to those three spatial positions and thus Fig. 4 is also representative of any two consecutive generic states, $k$ and $k + 1$.

The horizontal and vertical displacement components of the center of gravity are now computed by considering the elastic (bending) deformation of a simply-supported structure – Fig. 5. For the quasi-static model, the ground contact point for the leg that rolls is modeled by a pinned support while for the leg that drags a simple (sliding) support is used. The Castigliano’s first
A horizontal force $H$ and then a vertical one $V$ are applied at the center of gravity in the first half of a cycle. The incremental horizontal and vertical displacements of the center of gravity are:

$$
\Delta x^{(1)}_{k \rightarrow k+1} = \frac{1}{E_b} \left( \frac{1}{I_2^{(e)}} \delta h_1 + \frac{1}{I_1^{(e)}} \delta h_4 \right) + \frac{1}{E_a I_a + E_b \left( I_1^{(e)} + I_2^{(e)} \right)} \frac{L}{\theta_k} (C_{h1} \delta h_2 + \delta h_3),
$$

$$
\Delta y^{(1)}_{k \rightarrow k+1} = \frac{1}{E_b} \left( \frac{1}{I_2^{(e)}} \delta v_1 + \frac{1}{I_1^{(e)}} \delta v_4 \right) + \frac{1}{E_a I_a + E_b \left( I_1^{(e)} + I_2^{(e)} \right)} \frac{L}{\theta_k} (C_{h1} \delta v_2 + \delta v_3),
$$

(11)

where the new terms are given in Eqs. (A.2a)–(A.2e) and (A.3a)–(A.3d).

Eqs. (11) are also valid for the second phase of one cycle’s motion when the bending moment changes its direction (sign).

3.2.2. Rigid-body motion

The rigid-body components of the center of gravity displacement are produced by rotations of the entire structure around one leg’s ground contact point. The rotation is produced by the front
leg during the first phase of a cycle and then by the rear one during the second phase. Simple geometric considerations allow formulating the incremental displacement components of the center of gravity and rotation radius during the first phase as:

\[
\Delta x_{k \rightarrow k+1}^{(rb,1)} = 2R_{1k} \sin \frac{\alpha_{1,k+1} - \alpha_{1,k}}{2} \cos \left( \frac{\xi_{1,k} - \alpha_{1,k+1} - \alpha_{1,k}}{2} \right),
\]

\[
\Delta y_{k \rightarrow k+1}^{(rb,1)} = 2R_{1k} \sin \frac{\alpha_{1,k+1} - \alpha_{1,k}}{2} \sin \left( \frac{\xi_{1,k} - \alpha_{1,k+1} - \alpha_{1,k}}{2} \right),
\]

where

\[
R_{1k} = \sqrt{L^2 + 4 \left( \frac{L^2}{\theta_k} \sin^2 \frac{\theta_k}{4} - 2l_{1k} \frac{L}{\theta_k} \sin \frac{\theta_k}{4} \sin \left[ \alpha_{1,k} - \left( \beta_k + \frac{\theta_k}{4} \right) \right] \right)}.
\]

\[
\xi_{1,k} = \arcsin \left\{ \frac{2L}{\theta_k R_{1k}} \sin \frac{\theta_k}{4} \cos \left[ \alpha_{1,k} - \left( \beta_k + \frac{\theta_k}{4} \right) \right] \right\}.
\]

The same amounts are computed in a similar manner for the second phase, namely:

\[
\Delta x_{k \rightarrow k+1}^{(rb,2)} = 2R_{2k} \sin \frac{\alpha_{2,k+1} - \alpha_{2,k}}{2} \cos \left( \frac{\xi_{2,k} + \alpha_{2,k+1} - \alpha_{2,k}}{2} \right),
\]

\[
\Delta y_{k \rightarrow k+1}^{(rb,2)} = -2R_{2k} \sin \frac{\alpha_{2,k+1} - \alpha_{2,k}}{2} \sin \left( \frac{\xi_{2,k} + \alpha_{2,k+1} - \alpha_{2,k}}{2} \right),
\]

with

\[
R_{2k} = \sqrt{L^2 + 4 \left( \frac{L^2}{\theta_k} \sin^2 \frac{\theta_k}{4} - 2l_{2k} \frac{L}{\theta_k} \sin \frac{\theta_k}{4} \sin \left[ \alpha_{2,k} - \left( \beta_k + \frac{\theta_k}{4} \right) \right] \right)}.
\]

\[
\xi_{2,k} = \arcsin \left\{ \frac{2L}{\theta_k R_{2k}} \sin \frac{\theta_k}{4} \cos \left[ \beta_k - \left( \alpha_{2,k} + \frac{\theta_k}{4} \right) \right] \right\}.
\]

The total displacement components of the center of gravity are calculated by summing the compliant and rigid-body components, namely

\[
\Delta q_{k \rightarrow k+1}^{(i)} = \Delta q_{k \rightarrow k+1}^{(c,i)} + \Delta q_{k \rightarrow k+1}^{(rb,i)}, \quad q = x, y; \quad i = 1, 2.
\]

3.2.3. Maximum deformation angle of the unimorph

It is important to evaluate the maximum angular deformation of the unimorph produced by the bending moment since this will completely define the range \([\theta_0 \rightarrow \theta_1]\) over which the variable \(\theta\) is allowed to vary during one cycle of motion. This can be achieved by expressing the static deformation of the simply-supported structure of Fig. 5 during the first phase when the front leg rolls and the rear one drags.
The Castigliano’s first theorem is applied to compute \( \delta_{4h} \), the horizontal displacement of point 4 (Fig. 5), by means of a dummy horizontal load \( H_{40} \) \((H_{40} = 0)\) and by applying the maximum value of the bending moment produced by the unimorph, \( M_f \)

\[
\delta_{4h} = \delta^{(rb)}_{4h} + \frac{1}{E_b} \left( \frac{1}{I'_1} C'_{h5} + \frac{1}{I'_2} C'_{h6} \right),
\]

where \( \delta^{(rb)}_{4h} \), the horizontal displacement of a rigid-legs inchworm, is calculated by using Eqs. (11) and is of the form

\[
\delta^{(rb)}_{4h} = \frac{1}{E_a I_a + E_b (I_{b1} + I_{b2})} \frac{L}{\theta_0} \left[ C'_{h1} \theta_0 + C'_{h2} (1 - \cos \theta_0) + C'_{h3} \sin \theta_0 + \frac{C'_{h4} - \cos 2\theta_0}{2} \right].
\]

The new symbols in Eqs. (17) and (18) are given in Eqs. (A.4) and (A.5). The assumption is applied here that the unimorph tabs have zero length and therefore the curved beam has constant-thickness cross-section over its whole length.

Eqs. (17) and (18), together with Eqs. (A.4) and (A.5) illustrate that

\[
\delta_{4h} = f_\theta(M_f).
\]

Simple geometric considerations indicate that the horizontal displacement of the rear leg should comply with

\[
\delta_{4h} + l_{21} \sin \alpha'_{21} + L_e \cos \beta_1 - l_{11} \sin \alpha'_{11} = l_{20} \sin \alpha'_{20} + L_c \cos \beta_0 - l_{10} \sin \alpha'_{10}
\]

which shows that \( \theta_1 \) can be expressed as a function of \( \theta_0 \) and \( M_f \) only.

### 3.2.4. Dynamic model

The motion has been analyzed so far by only considering the geometry of deformation and rigid-body motion, which enabled to formulate a single-degree of freedom model. Inertia and time are now included into a lumped-parameter model that is used to derive the dynamic equations of motion for the inchworm center of gravity with respect to a fixed reference frame as shown in Fig. 4. The interval \( \theta_0, \theta_1 \) is univocally related to the time interval \( t_0, t_1 \). The mass of the legs is neglected and the mass of the system is concentrated at the center of gravity. Since the clamps have masses that are approximately equal and the angle \( \theta \) is relatively small, it is considered that the center of gravity is placed at the unimorph midpoint.

Newton’s second law is applied for the \( x, y \) and \( z \) axes and the corresponding equations are:

\[
M \ddot{x}^{(1)} = \mu \text{GRF}_1^{(1)} - \mu \text{GRF}_2^{(1)},
\]

\[
M \ddot{y}^{(1)} = \text{GRF}_1^{(1)} + \text{GRF}_2^{(1)} - Mg,
\]

\[
2M \frac{L^2}{\theta_0^2} \ddot{\theta}^{(1)} = \text{GRF}_1^{(1)} \left( l_{11} \sin \alpha'_{11} + \frac{L_e}{2} \cos \beta_k \right) - \text{GRF}_2^{(1)} \left( l_{21} \sin \alpha'_{21} + \frac{L_e}{2} \cos \beta_k \right)
\]

\[
- \mu \text{GRF}_1^{(1)} \left( l_{11} \cos \alpha'_{11} + \frac{L_e}{2} \sin \beta_k \right) - \mu \text{GRF}_2^{(1)} \left( l_{21} \cos \alpha'_{21} + \frac{L_e}{2} \sin \beta_k \right)
\]

for the first half of a cycle. A similar equation can be written for the second half by changing the sign of the friction force in Eqs. (21).
The central difference scheme is now utilized to express the acceleration in terms of displacement components

\[
\ddot{q}_k = \frac{q_{k+1} - 2q_k + q_{k-1}}{\Delta t^2}, \quad q = x, y, \beta; \quad k = 1 \rightarrow n,
\]  

(22)

where \( \Delta t \) is the time step.

Eqs. (21) are discretized with respect to time at the station \( k \) and their solutions are:

\[
\text{GRF}^{(i)}_{1,k} = \frac{D^{(i)}_{1,k}}{D^{(i)}_{2,k}} Mg
\]

\[
\text{GRF}^{(i)}_{2,k} = \frac{D^{(i)}_{3,k}}{D^{(i)}_{2,k}} Mg
\]

\[
\Delta t^{(i)}_k = \sqrt{\frac{d^{(i)}_{1,k}}{(-1)^{i+1}\text{GRF}^{(i)}_{1,k} + (-1)^i\text{GRF}^{(i)}_{2,k}}}, \quad i = 1, 2.
\]  

(A.6a)–(A.6c) gives the new amounts of Eqs. (23).

The time step might vary as indicated by Eqs. (23). In order to ensure numerical accuracy, the minimum value was taken to be the actual time step.

4. Numerical simulation and design

Preliminary simulation was first performed in order to select the geometry configuration that is capable of maximizing the robot performance. An inchworm prototype was designed based on this data. Subsequent numerical simulation addressed the ground reaction forces and the horizontal velocity for the prototype’s center of gravity.

4.1. Preliminary simulation

The purpose was to enable primary selection of the geometric parameters that could maximize the deformation angle of the unimorph, \( \theta_1 \) and the horizontal displacement of the inchworm center of gravity, \( \Delta x \).

The following conclusions were derived with reference to Fig. 6(a)–(e):

- The parameters \( r_1, r_2, H_1, H_2, h_1 \) and \( h_2 \) have little influence on \( \theta_1 \), as illustrated in Fig. 6(a) and (b), for instance.
- The inclination angle of the front leg, \( \alpha_{10} \), should be small (approaching zero, as indicated by Fig. 6(a)) while the inclination angle of the rear leg, \( \alpha_{20} \), should be large, as suggested by Fig. 6(b).
- The angles \( \delta \) and \( \gamma \) that define the unimorph-legs relative position need to be small as shown in Fig. 6(c).
- Fig. 6(d) indicates that more mass is needed to the rear of the device.
Fig. 6. Maximum deformation angle, $\theta_1$, in terms of: (a) $H_1$ and $a_{10}$; (b) $H_2$ and $a_{20}$; (c) $\gamma$ and $\delta$; (d) $m_1$ and $m_2$; (e) $c_2$ and $t_2$. 
• For values in a practical range, the legs cross-sectional parameters have little influence on $\theta_1$, except for small values that are likely to increase $\theta_1$, as illustrated in Fig. 6(e) for instance. Similar conclusions were drawn with regard to the horizontal displacement of the robot’s center of gravity.

4.2. Actual design of an inchworm prototype

An inchworm prototype was designed, as illustrated in Fig. 7, based on numerical simulation data and using a TH 1-R unimorph. The parameters defining the prototype are given in Table 1.

The inchworm displayed a pattern of self-controlled falling-ahead motion at a speed of approximately 0.001 m/s when a 45 zero-to-peak sine-voltage was applied to the unimorph, at approximately 37 Hz.

The device produced a horizontal motion consisting of two-segments over one cycle: a smaller one, during the first half and a bigger one, during the second, as predicted by the simplified analysis of motion.

4.3. Dynamic model simulations

Data of the actual design was used in conjunction with the dynamic model in order to investigate the ground reaction forces and the horizontal velocity of the robot center of gravity.

Evaluation of the ground reaction forces, GRF$_1$ and GRF$_2$ allowed to note the following remarks:

![Fig. 7. Inchworm prototype.](image)

<p>| Table 1 |
| Parameters of the actual inchworm design ($\gamma = 100^\circ, \delta = 90^\circ, d_{31} = 9 \times 10^{-11}$ (m/V)) |</p>
<table>
<thead>
<tr>
<th>$h$ (m) $\times 10^{-3}$</th>
<th>$H$ (m) $\times 10^{-3}$</th>
<th>$b$ (m) $\times 10^{-3}$</th>
<th>$r$ (m) $\times 10^{-3}$</th>
<th>$x_0$ (deg)</th>
<th>$c$ (m) $\times 10^{-3}$</th>
<th>$t$ (m) $\times 10^{-3}$</th>
<th>$m$ (kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Front leg</td>
<td>8</td>
<td>12</td>
<td>5</td>
<td>4</td>
<td>5</td>
<td>8</td>
<td>0.1</td>
</tr>
<tr>
<td>Rear leg</td>
<td>10</td>
<td>14</td>
<td>8</td>
<td>4</td>
<td>5</td>
<td>12</td>
<td>0.1</td>
</tr>
</tbody>
</table>
• The model-predicted values of the ground reaction forces are almost uninfluenced by the angle $\theta$ for the rear leg during the whole cycle, as illustrated in Fig. 8(a) and (b).
• The ground reaction force on the front leg is larger than the one on the rear leg during the first part of a cycle and vice versa during the second part, as indicated by Fig. 8(a) and (b).

The horizontal velocity of the center of gravity was calculated for a time step of 0.001 (s) by means of the central difference equation

$$v_x = \frac{x_{k+1} - x_k}{\Delta t}.$$  \hfill (24)

Fig. 8. Ground reaction forces during: (a) the first part of a motion’s cycle; (b) the second part of a motion’s cycle.

Fig. 9. Horizontal velocity of the center of gravity, $v_x$, during: (a) the first part of a motion’s cycle; (b) the second part of a motion’s cycle.
The model results were in agreement with the velocity data of the inchworm prototype (of approximately 0.001 m/s), as illustrated in Fig. 9(a) and (b).

5. Conclusions

The mathematical model and actual design of an inchworm locomotion device are presented. The lightly damped structure consists of a unimorph piezoelectric actuator and two custom-designed compliant legs. The resonant excitation of the inchworm results in a two-part unidirectional falling-ahead motion over horizontal smooth terrain. A mathematical model is developed that includes formulation of the actuator forced bending, geometrical analysis of the motion and evaluation of the ground reaction forces and horizontal velocity. Preliminary simulations that were based on the geometry of deformation allowed selection of the geometric parameters for designing an inchworm prototype. Subsequent numerical simulation used the dynamic model to evaluate the ground reaction forces and horizontal velocity. The experimental and model velocity data were in good agreement.

Acknowledgements

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Appendix

\[ C_\ell = \frac{C_{\ell_1}}{E_a C_{\ell_2} + E_b C_{\ell_3}}, \]

\[ C_{\ell_1} = 28 A_a E_a E_b [A_{b1}(h_a + h_{b1}) - A_{b2}(h_a + h_{b2})] C_{f_4}, \]

\[ C_{\ell_2} = A_a \left\{ A_{b1} E_b (h_a^2 + 15h_a h_{b1} + 14h_{b1}^2) + 14 \left[ A_{b2} E_b (h_a + h_{b2})^2 + 4C_{f_4} \right] \right\}, \]

\[ C_{\ell_3} = A_{b1} \{ A_{b2} E_b (h_{b2} - h_{b1}) [15h_a + 14(h_{b1} + h_{b2})] \} + 56 (A_{b1} - A_{b2}) C_{f_4}, \]

\[ C_{f_4} = E_a I_a + E_b (I_{b1} + I_{b2}), \]

\[ \delta_{h1} = -C_{h1} C_{h2} \sin^2 \varphi_{2,k} \frac{l_{2,k}^3}{3}, \]

\[ \delta_{h2} = A_{21} A_{23} \frac{\theta_k}{2} + (A_{21} A_{24} + A_{22} A_{23}) \left( 1 - \cos \frac{\theta_k}{2} \right) + \frac{A_{22} A_{24}}{4} (\theta_k - \sin \theta_k), \]
\[ \delta_{h3} = A_{21} A_{33} \frac{\theta_k}{2} - (A_{21} A_{34} + A_{22} A_{33}) \left( \cos \theta_k - \cos \frac{\theta_k}{2} \right) - \frac{A_{22} A_{24}}{4} \cos \left( \beta_k - \frac{\theta_k}{2} \right) (\cos 2\theta_k - \cos \theta_k) \]
\[ + \frac{A_{22}}{4} \left( \theta_k - \sin 2\theta_k + \sin \theta_k \right) \left[ A_{34} - A_{24} \sin \left( \beta_k - \frac{\theta_k}{2} \right) \right] \]
\[ + A_{21} A_{24} \left[ \cos \left( \beta_k - \frac{\theta_k}{2} \right) \left( \cos \theta_k - \cos \frac{\theta_k}{2} \right) + \sin \left( \beta_k - \frac{\theta_k}{2} \right) \left( \sin \theta_k - \sin \frac{\theta_k}{2} \right) \right] , \]
\[ \delta_{h4} = l_{1,k} (A_{41} A_{43}) + \frac{A_{41} A_{44} + A_{42} A_{43}}{2} l_{1,k} + \frac{A_{42} A_{44}}{3} l_{1,k}^2 , \]

(A.2a)

\[ A_{21} = m_{1,k} - C_{h2} l_{2,k} \sin \xi'_{2,k} , \]
\[ A_{22} = m_2 g \frac{L}{\theta_k} - C_{h2} \frac{L}{\theta_k} , \]
\[ A_{23} = l_{2,k} \sin \xi'_{2,k} , \]
\[ A_{24} = \frac{L}{\theta_k} , \]
\[ A_{33} = C_{h1} l_{2,k} \sin \xi'_{2,k} - \frac{L}{\theta_k} \cos \beta_k , \]
\[ A_{34} = C_{h1} \frac{L}{\theta_k} , \]
\[ A_{41} = m_2 g L_{c,k} \cos \beta_k - C_{h1} (l_{2,k} \sin \xi'_{2,k} + L_{c,k} \cos \beta_k) , \]
\[ A_{42} = [C_{h2} - (m_1 + m_2) g] \sin \xi'_{1,k} , \]
\[ A_{43} = C_{h1} \left( l_{2,k} \sin \xi'_{2,k} + L_{c,k} \cos \beta_k \right) - \frac{L}{\theta_k} \left[ \cos \beta_k + \cos \left( \beta_k + \frac{\theta_k}{2} \right) \right] , \]
\[ A_{44} = -(C_{h1} \sin \xi'_{1,k} + \cos \xi'_{1,k}) , \]

(A.2b)

\[ C_{h1} = \frac{(L/\theta_k) \left[ \cos \beta_k - \cos (\beta_k + (\theta_k/2)) \right] + l_{1,k} \cos \xi'_{1,k}}{l_{c,k} \cos \beta_k + l_{2,k} \sin \xi'_{2,k} - l_{1,k} \sin \xi'_{1,k}} , \]
\[ C_{h2} = \frac{m_2 L_{c,k} \cos \beta_k - (m_1 + m_2) l_{1,k} \sin \xi'_{1,k}}{l_{c,k} \cos \beta_k + l_{2,k} \sin \xi'_{2,k} - l_{1,k} \sin \xi'_{1,k}} g , \]

(A.2c)
\[\delta v_1 = -C_h C_v^3 \sin^2 \alpha_{2,k} \frac{l_{2,k}^3}{3},\]

\[\delta v_2 = \delta h_2,\]

\[\delta v_3 = A_{21} A_{36} \frac{\theta_k}{2} - (A_{21} A_{37} + A_{22} A_{36}) \left( \cos \theta_k - \cos \frac{\theta_k}{2} \right) + \frac{A_{22} A_{24}}{4} \sin \left( \beta_k - \frac{\theta_k}{2} \right) \left( \cos 2\theta_k - \cos \theta_k \right) + \frac{A_{22}}{4} \left( \theta_k - \sin 2\theta_k + \sin \theta_k \right) \left[ A_{37} - A_{24} \cos \left( \beta_k - \frac{\theta_k}{2} \right) \right] \]

\[\delta v_4 = l_{1,k} \left( A_{41} A_{45} + \frac{A_{41} A_{46} + A_{42} A_{45}}{2} l_{1,k} + \frac{A_{42} A_{46}}{3} l_{1,k}^2 \right),\]

(A.3a)

\[A_{36} = C_h^1 l_{2,k} \sin \alpha'_{2,k} + \frac{L}{\theta_k} \cos \beta_k,\]

A.3b

\[A_{37} = C_h^1 \frac{L}{\theta_k},\]

A.3c

\[A_{45} = C_v^3 \left( l_{2,k} \sin \alpha'_{2,k} + L \cos \beta_k \right) - \frac{L}{\theta_k} \left[ \sin \left( \beta_k + \frac{\theta_k}{2} \right) - \sin \beta_k \right],\]

\[A_{46} = (1 - C_v^3) \sin \alpha'_{1,k},\]

A.3c

\[C_v^3 = \frac{(L/\theta_k) \left[ \sin \left( \beta_k + (\theta_k/2) \right) - \sin \beta_k \right] - l_{1,k} \sin \alpha'_{1,k}}{L \cos \beta_k + l_{2,k} \sin \alpha'_{2,k} - l_{1,k} \sin \alpha'_{1,k}},\]

(A.3d)

\[C_h^1 = (M_f - V_{40} l_{20} \sin \alpha_{20}) \left( l_{20} \cos \alpha_{20} - \frac{L}{\theta_0} \right),\]

\[C_h^2 = (m_2 g - V_{40}) \frac{L}{\theta_0} \left( l_{20} \cos \alpha_{20} - \frac{L}{\theta_0} \right),\]

\[C_h^3 = (M_f - V_{40} l_{20} \sin \alpha_{20}) \frac{L}{\theta_0},\]

(A.4)

\[C_h^4 = \frac{1}{2} (m_2 g - V_{40}) \frac{L^2}{\theta_0^2},\]
\[
C_{h5}' = l_10 \left( C_{h7}' C_{h8}' + \frac{C_{h7}' C_{h9}' - C_{h8}' \cos \alpha_{10}' l_10 - \frac{C_{h9}' \cos \alpha_{10}' l_10^2}{3}}{2} \right),
\]
\[
C_{h6}' = -\frac{p_{20}}{6} V_{40} \sin \alpha_{20}',
\]
\[
C_{h7}' = l_{20} \cos \alpha_{20}' - L_{c0} \sin \beta_0,
\]
\[
C_{h8}' = -(V_{40} + m_2 g) C_{h10}',
\]
\[
C_{h9}' = (V_{40} - m_1 g) \sin \alpha_{10}',
\]
\[
C_{h10}' = l_{20} \sin \alpha_{20}' + L_{c0} \cos \beta_0,
\]
\[
D_{1,k}^{(1)} = d_{1,k}^{(1)} \left( d_{5,k}^{(1)} - d_{1,k}^{(1)} d_{4,k}^{(1)} \right),
\]
\[
D_{2,k}^{(1)} = \left( d_{1,k}^{(1)} + d_{2,k}^{(1)} \right) \left( d_{5,k}^{(1)} - d_{1,k}^{(1)} d_{3,k}^{(1)} \right) + \left( d_{1,k}^{(1)} + d_{2,k}^{(1)} \right) \left( d_{5,k}^{(1)} - d_{1,k}^{(1)} d_{3,k}^{(1)} \right),
\]
\[
D_{3,k}^{(1)} = d_{1,k}^{(1)} \left( d_{5,k}^{(1)} - d_{1,k}^{(1)} d_{3,k}^{(1)} \right),
\]
\[
D_{1,k}^{(2)} = d_{1,k}^{(2)} \left( d_{5,k}^{(2)} + d_{1,k}^{(2)} d_{4,k}^{(2)} \right),
\]
\[
D_{2,k}^{(2)} = \left( d_{1,k}^{(2)} + d_{2,k}^{(2)} \right) \left( d_{5,k}^{(2)} + d_{1,k}^{(2)} d_{3,k}^{(2)} \right) + \left( d_{1,k}^{(2)} + d_{2,k}^{(2)} \right) \left( d_{5,k}^{(2)} + d_{1,k}^{(2)} d_{3,k}^{(2)} \right),
\]
\[
D_{3,k}^{(2)} = d_{1,k}^{(2)} \left( d_{5,k}^{(2)} + d_{1,k}^{(2)} d_{3,k}^{(2)} \right),
\]
\[
d_{1,i}^{(i)} = \frac{M}{\mu} \left( x_{k+1}^{(i)} - 2x_k^{(i)} + x_{k-1}^{(i)} \right),
\]
\[
d_{2,i}^{(i)} = M \left( y_{k+1}^{(i)} - 2y_k^{(i)} + y_{k-1}^{(i)} \right),
\]
\[
d_{3,i}^{(i)} = l_{1,i} \sin \alpha_{1,i} + \frac{L_{c,i}}{2} \cos \beta_i - \mu \left( l_{1,i} \cos \alpha_{1,i} + \frac{L_{c,i}}{2} \sin \beta_i \right),
\]
\[
d_{4,i}^{(1)} = l_{2,i} \sin \alpha_{2,i} + \frac{L_{c,i}}{2} \cos \beta_i + \mu \left( l_{2,i} \cos \alpha_{2,i} - \frac{L_{c,i}}{2} \sin \beta_i \right),
\]
\[
d_{4,i}^{(2)} = l_{2,i} \sin \alpha_{2,i} + \frac{L_{c,i}}{2} \cos \beta_i + \mu \left( l_{2,i} \cos \alpha_{2,i} - \frac{L_{c,i}}{2} \sin \beta_i \right),
\]
\[
d_{5,i} = I(\beta_{k+1} - 2\beta_k + \beta_{k-1}), \quad i = 1, 2.
\]


